Elias-Fano Encoding

A powerful tool for data structure design

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Tokyo, 10/04/2018

Problem

Consider a sequence S[0,n) of n *positive* and *monotonically increasing integers*, i.e., S[i-1] ≤ S[i] for 1 ≤ i ≤ n-1, possibly repeated.

How to represent it as a *bit vector* in which each original integer is *self-delimited*, using as few as possible bits?

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Huge research corpora describing different space/time trade-offs.

- Elias gamma/delta [Elias-1974]
- Variable Byte [Salomon-2007]
- Varint-G8IU [Stepanov et al.-2011]
- Simple-9/16 [Anh and Moffat 2005-2010]
- PForDelta (PFD) [Zukowski et al.-2006]
- OptPFD [Yan et al.-2009]
- Binary Interpolative Coding [Moffat and Stuiver-2000]

Inverted Indexes

Given a *textual collection* D, each document can be seen as a (multi-)set of terms. The set of terms occurring in D is the *lexicon* T.

For each term t in T we store in a list L_t the identifiers of the documents in which t appears.

The collection of all inverted lists $\{L_{t_1},\ldots,L_{t_T}\}$ is the inverted index.

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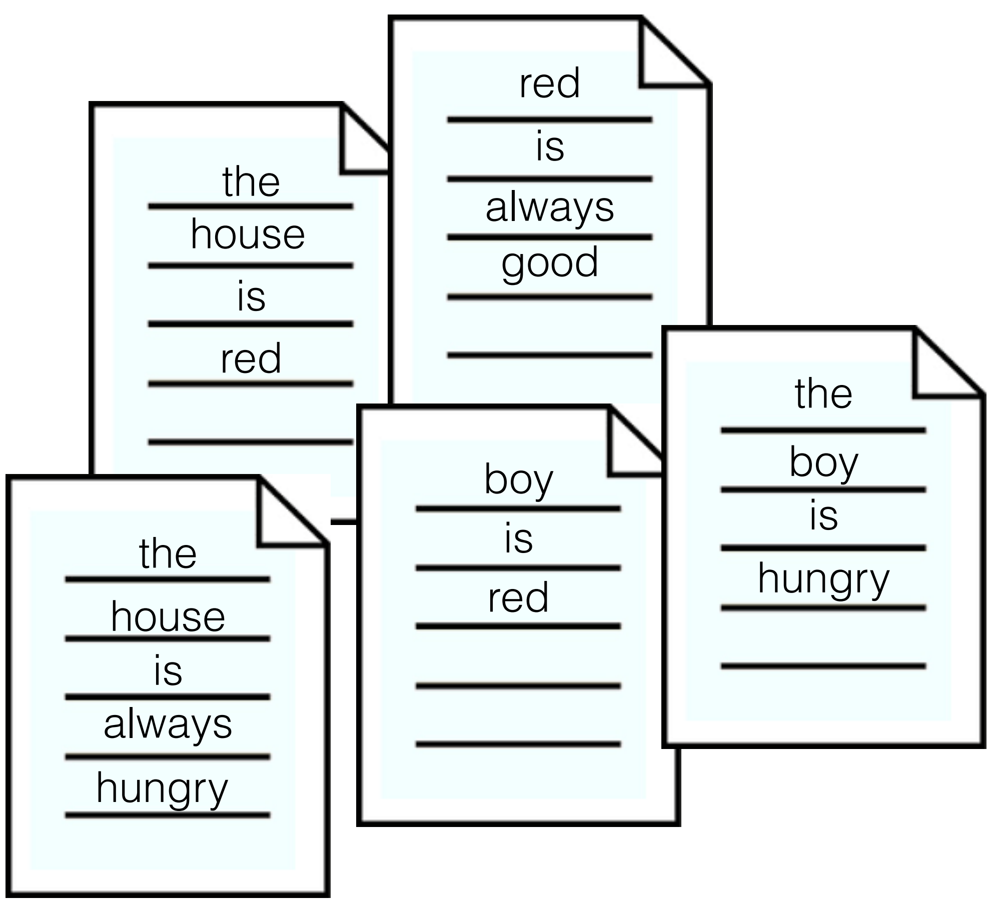

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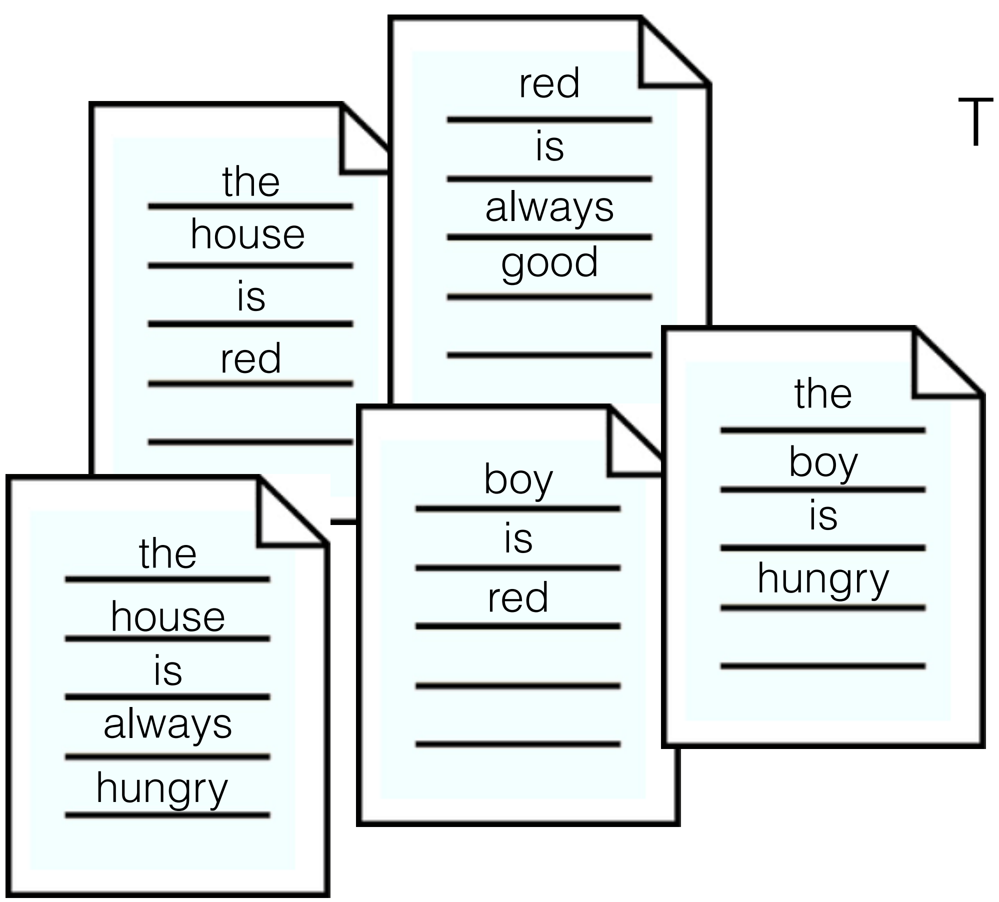

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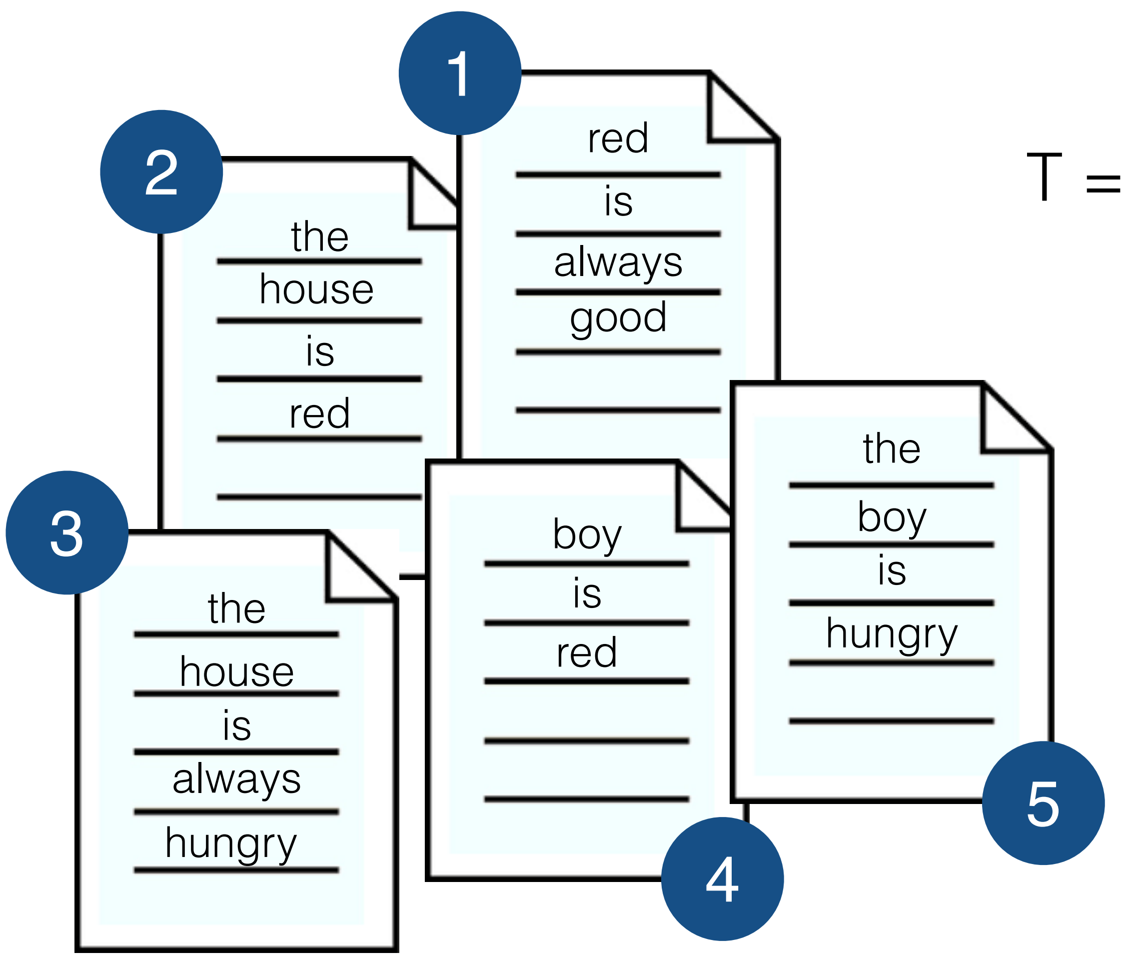

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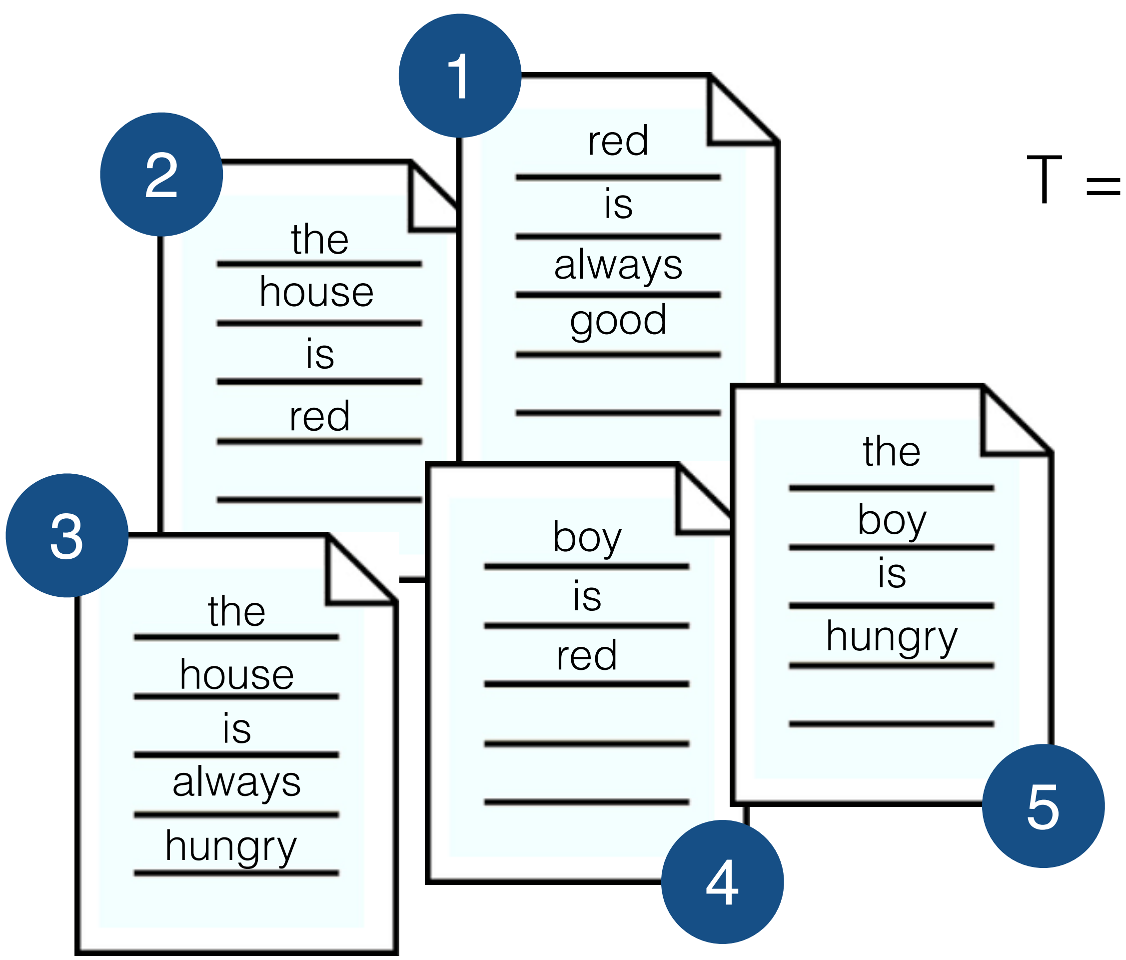

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Inverted Indexes owe their popularity to the *efficient resolution of queries*, such as: “return me all documents in which terms $\{t_1,\ldots,t_k\}$ occur”.

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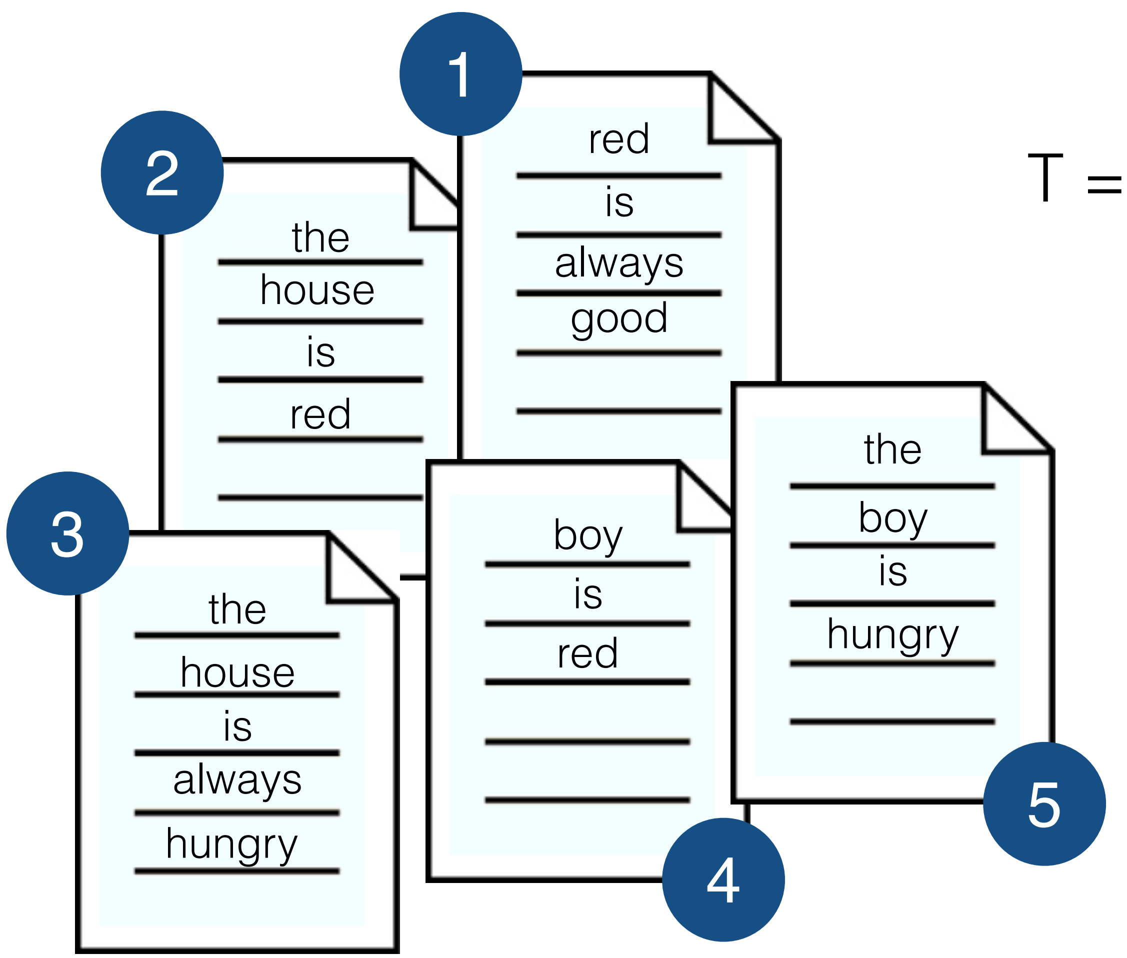

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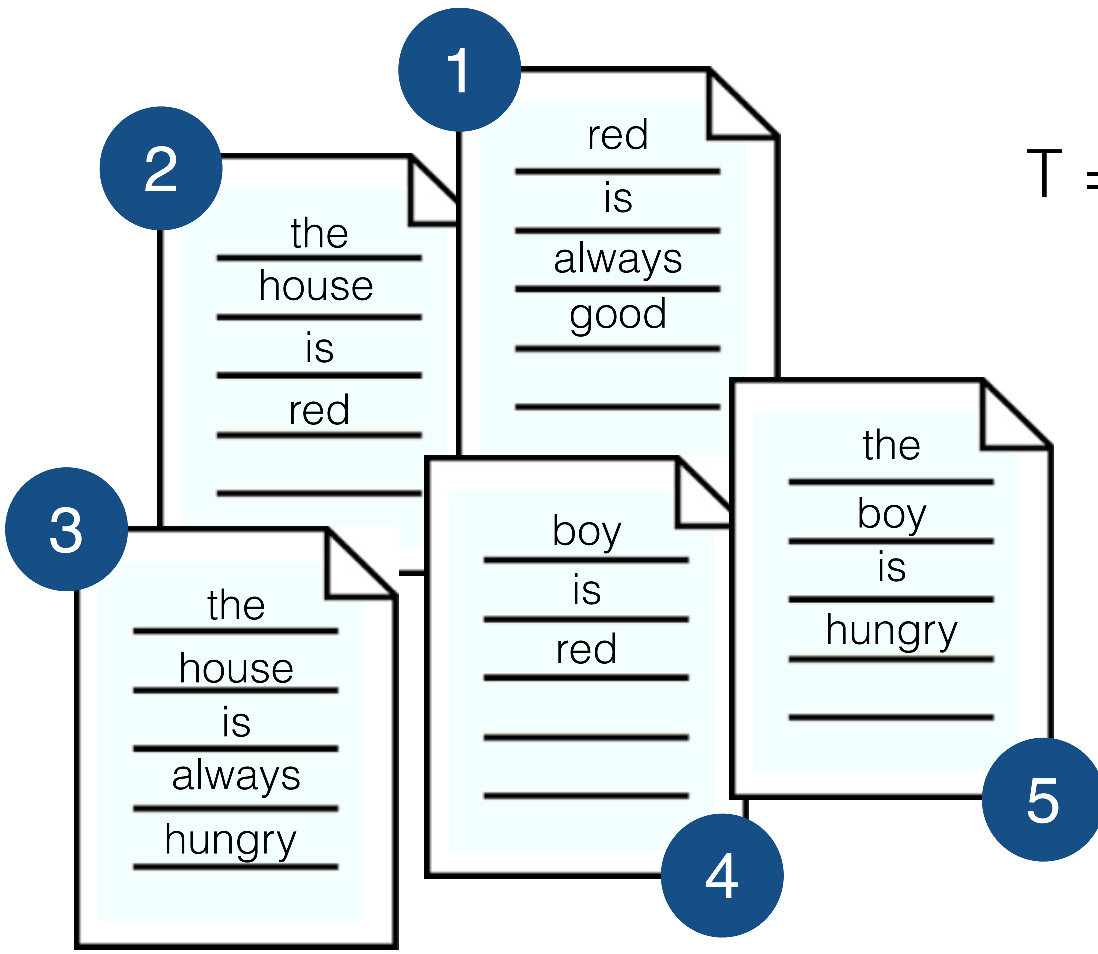

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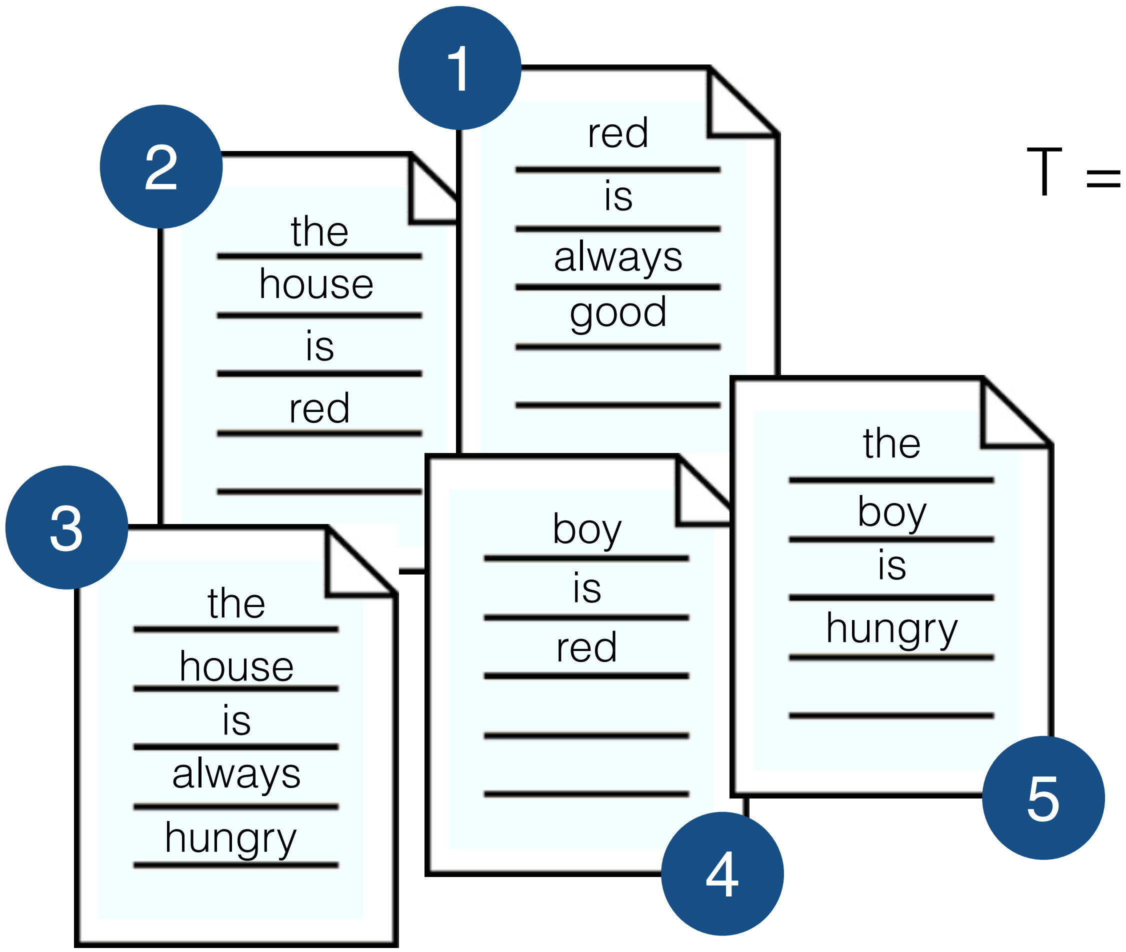

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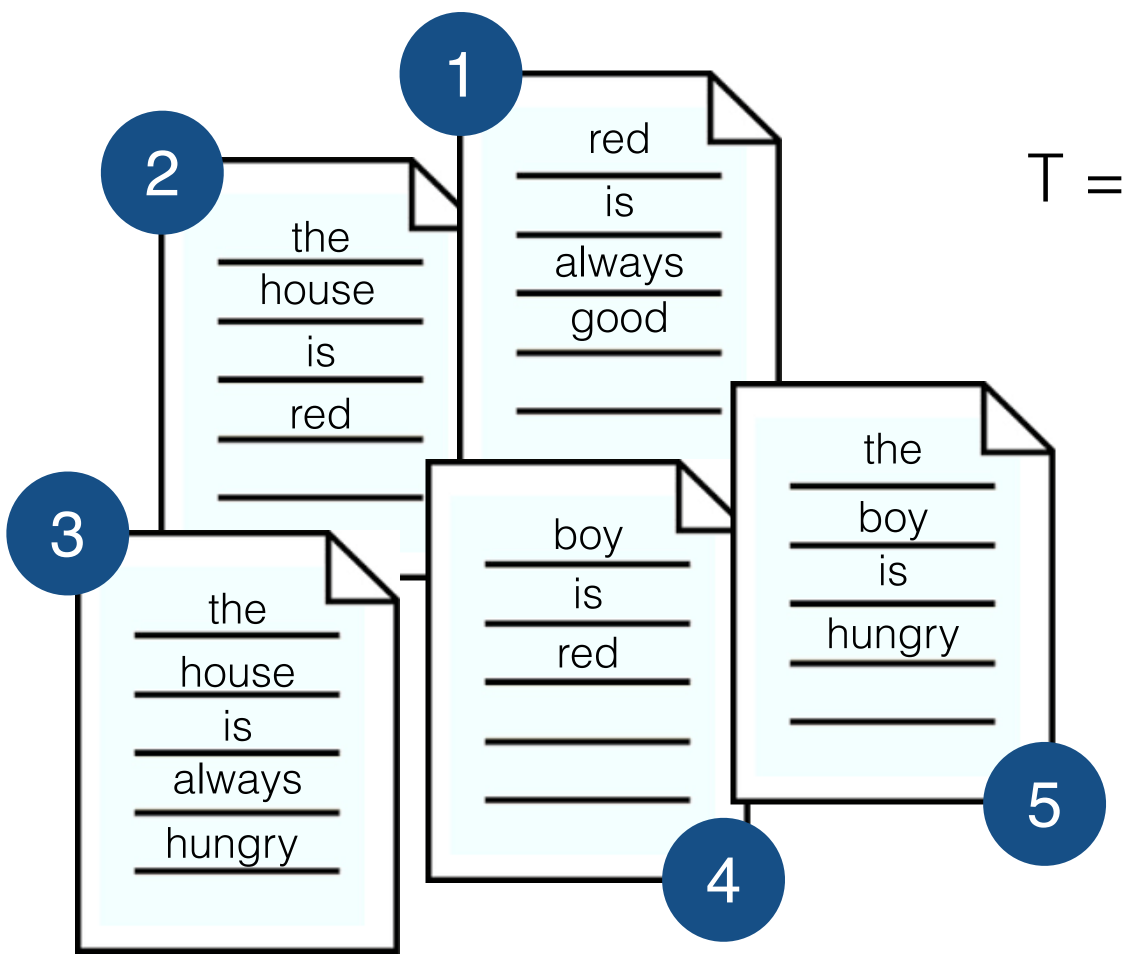

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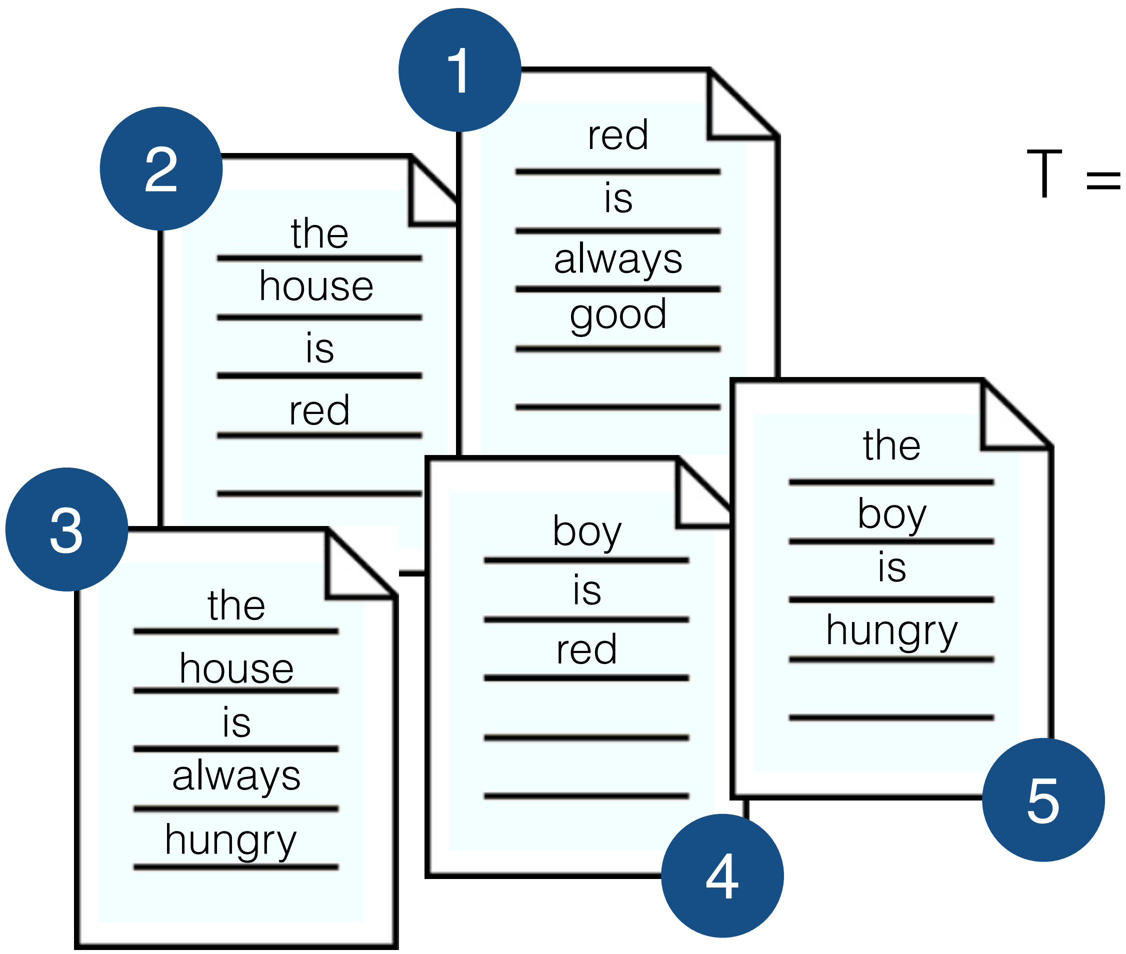

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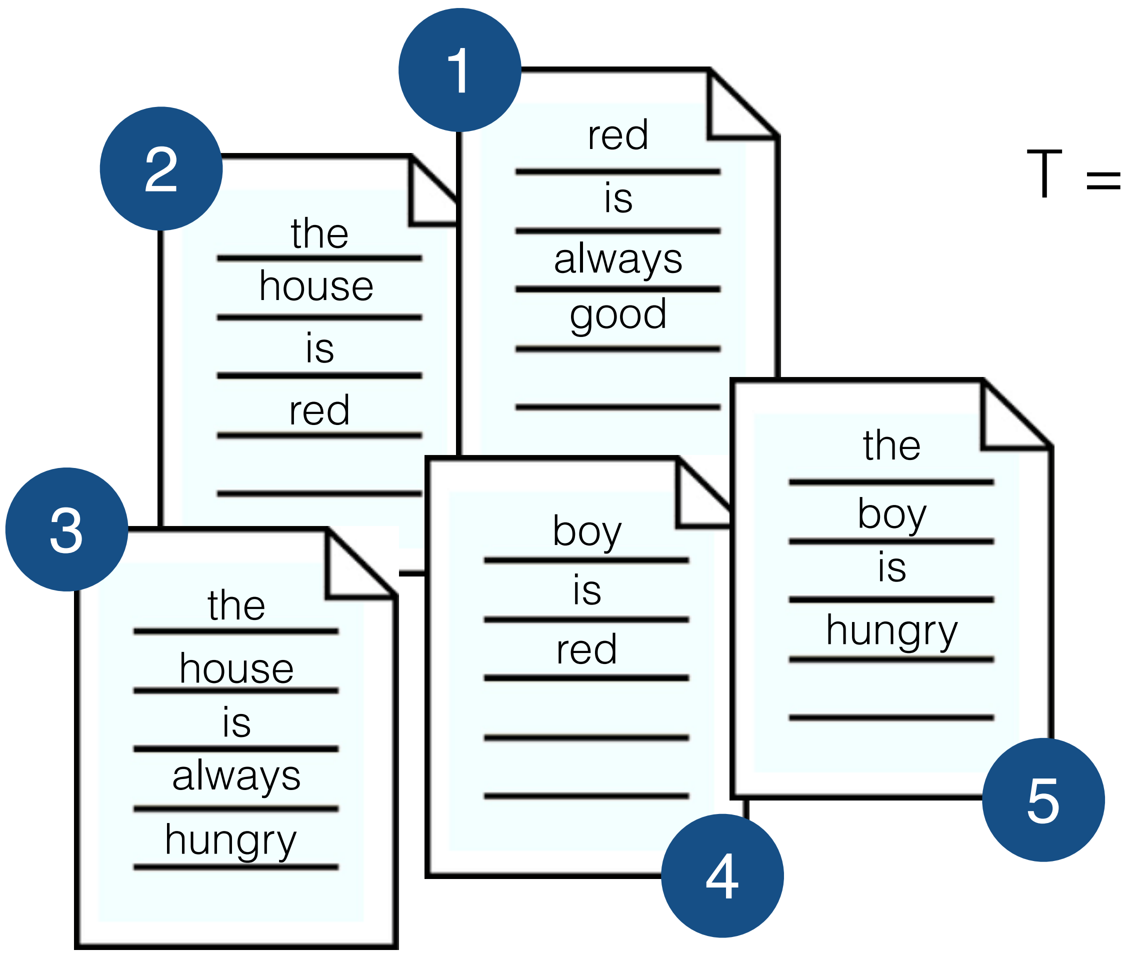

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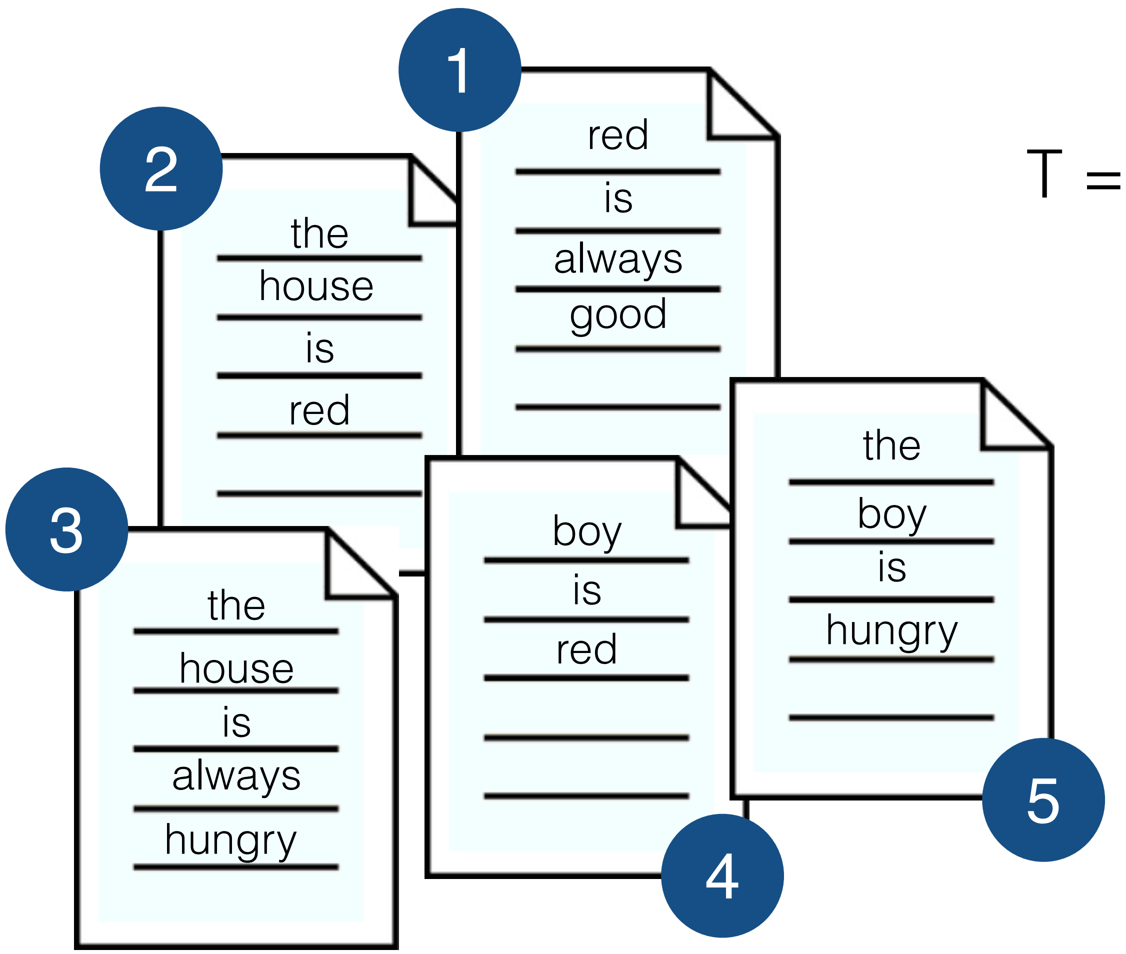

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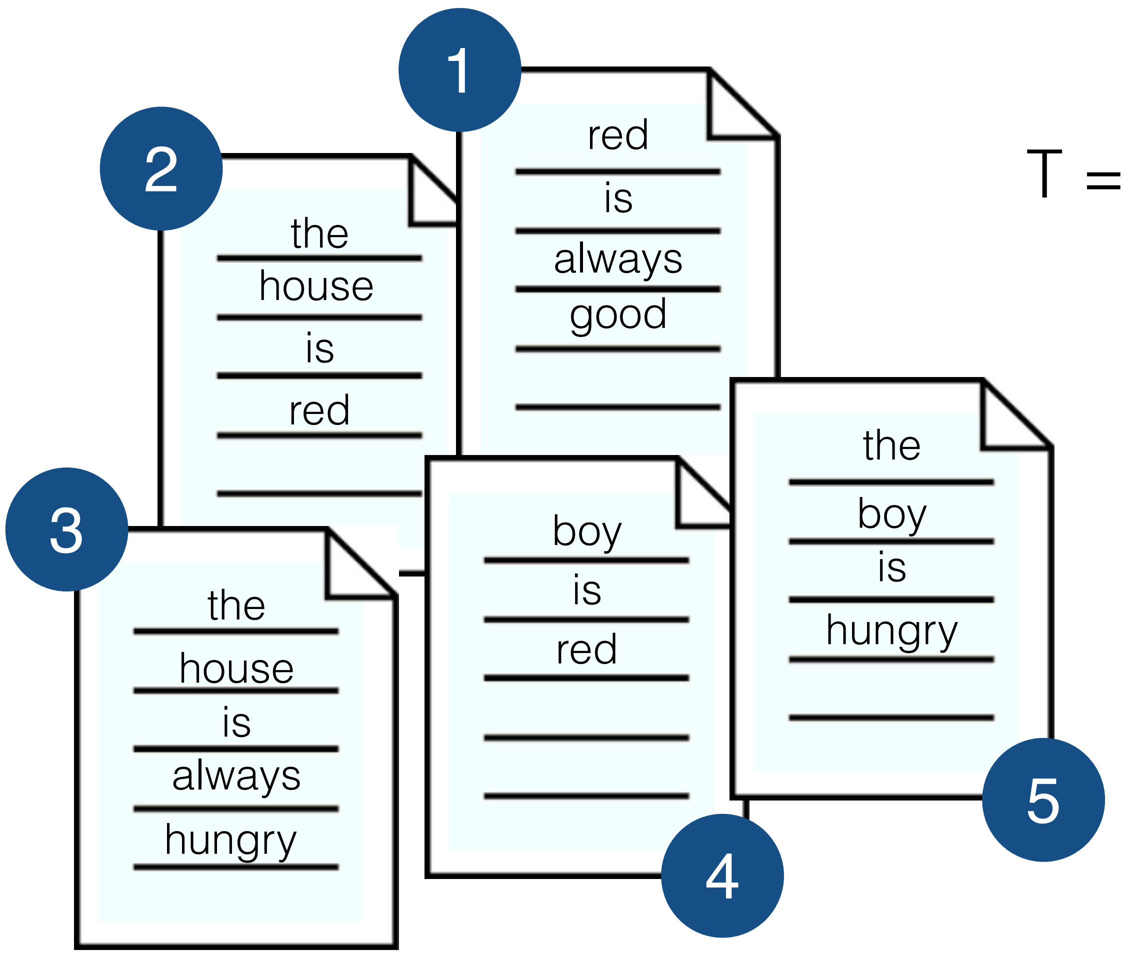

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intersection

Genesis - 1970s

Peter Elias
[1923 - 2001]

Robert Fano
[1917 - 2016]

Robert Fano. *On the number of bits required to implement an associative memory*. Memorandum 61, Computer Structures Group, MIT (1971).

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40 years later!

Sebastiano Vigna. *Quasi-succinct indices*.

In Proceedings of the 6-th ACM International Conference on Web Search and Data Mining (WSDM), 83-92 (2013).

Elias-Fano Encoding

3	1
4	2
7	3
13	4
14	5
15	6
21	7
43	8

Elias-Fano Encoding

	3	1
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u =	43	8

Elias-Fano Encoding

0 0 0 0 1 1	3	1
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Elias-Fano Encoding

low

$r = \lceil \log(u/n) \rceil$

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0 1 0	1 0 1	21	7
1 0 1	0 1 1	u = 43	8

L = 011100111101110111101011

Elias-Fano Encoding

high $\lceil \log u \rceil - r$	low $r = \lceil \log(u/n) \rceil$		
0 0 0	0 1 1	3	1
0 0 0	1 0 0	4	2
0 0 0	1 1 1	7	3
0 0 1	1 0 1	13	4
0 0 1	1 1 0	14	5
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1 0 1	0 1 1	$u =$ 43	8

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	high	low		
	$\lceil \log u \rceil - r$	$r = \lceil \log(u/n) \rceil$		
	0 0 0	0 1 1	3	1
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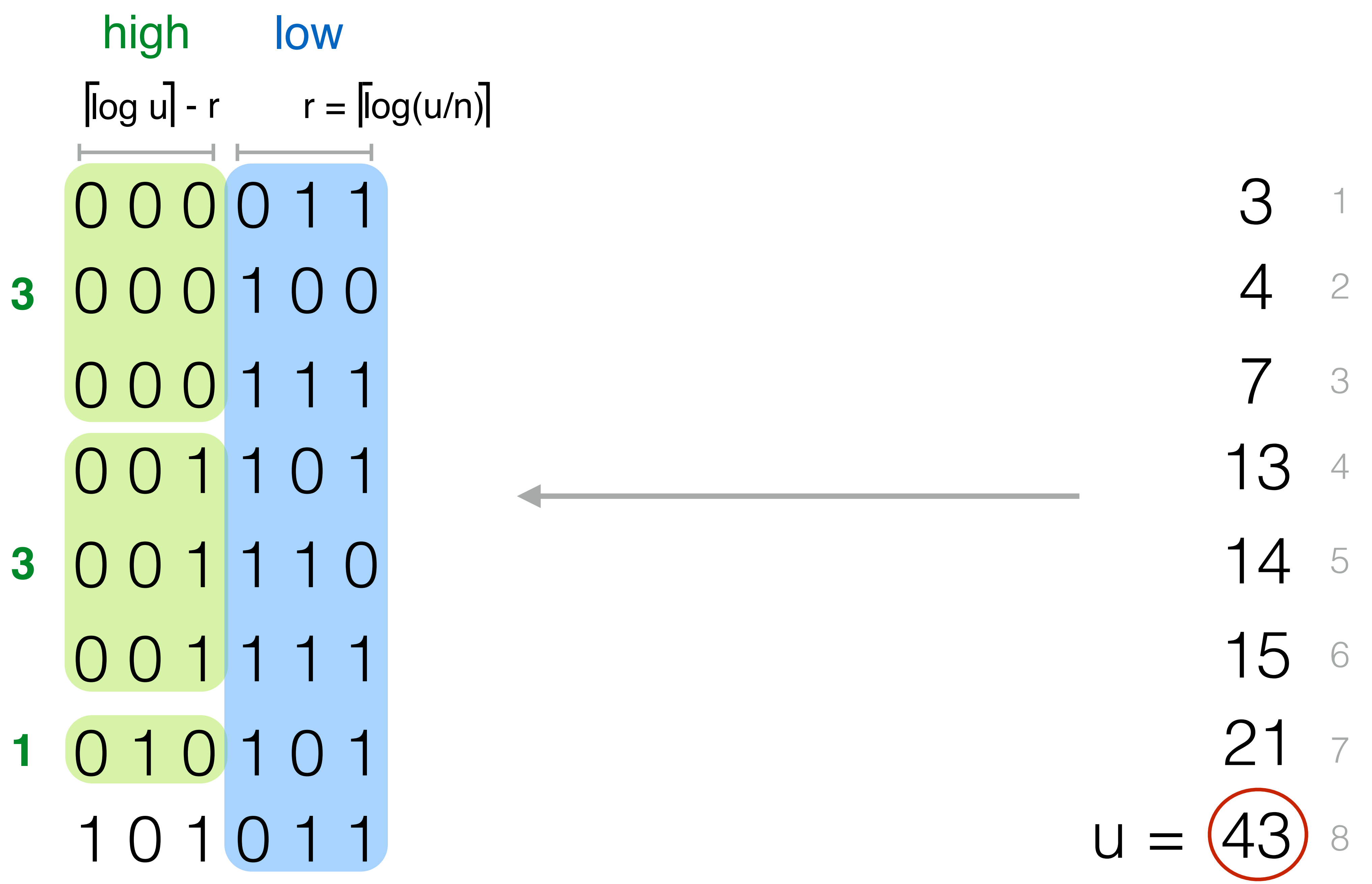

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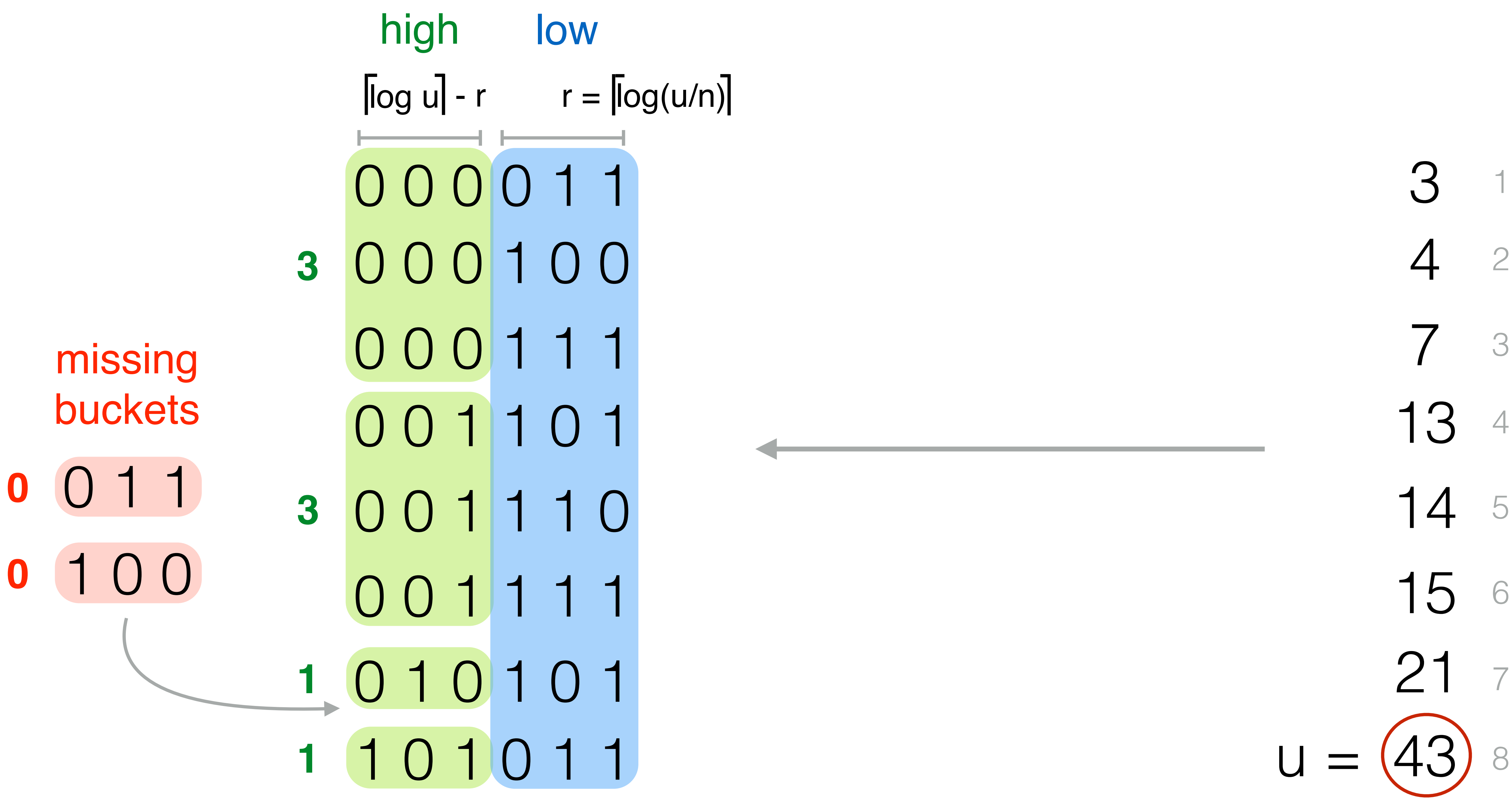

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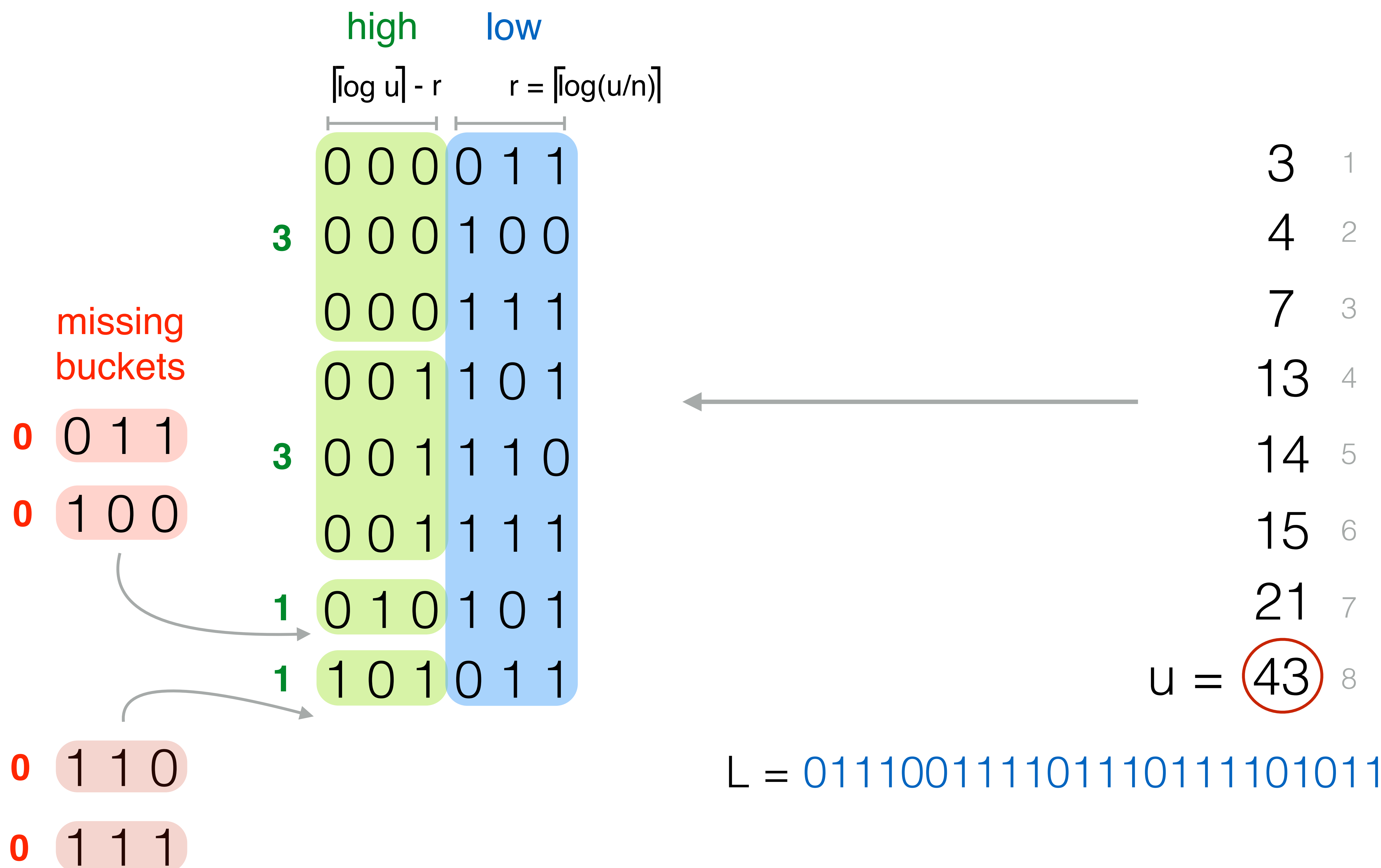

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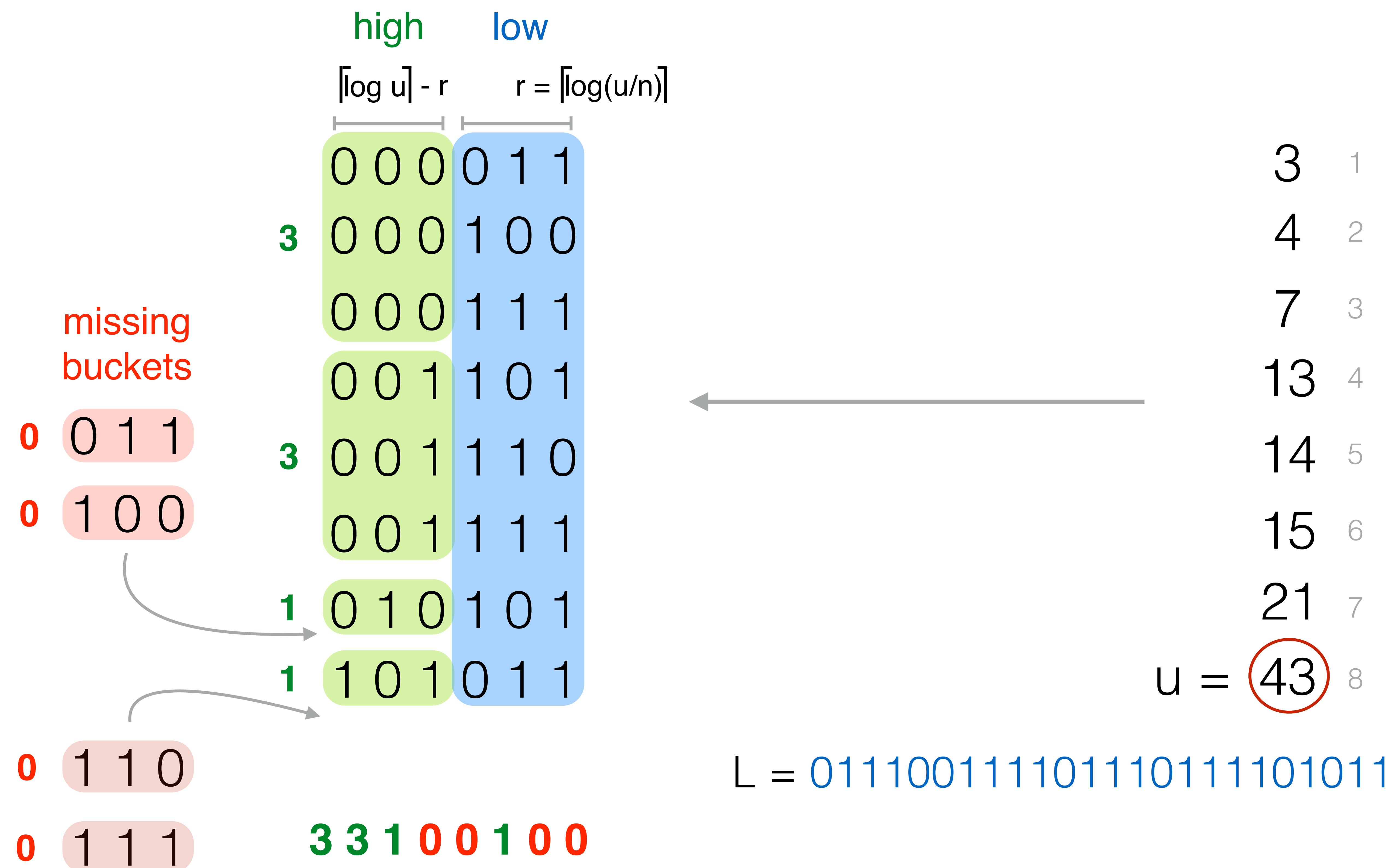

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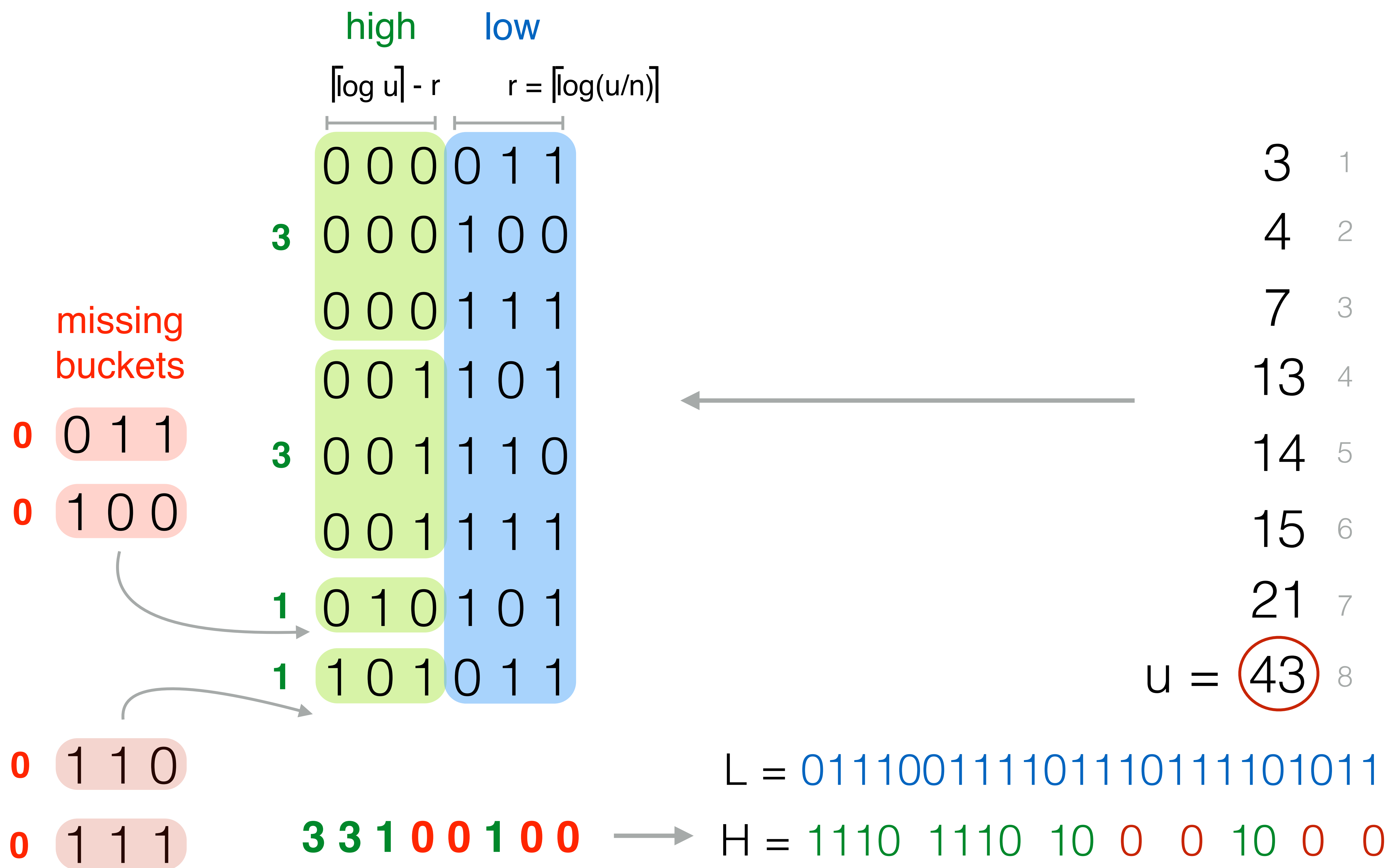

Properties - Space

$EF(S[0,n)) = ?$

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$\lceil \log(u/n) \rceil$

L = 0111001111011101111101011

H = 1110 1110 10 0 0 10 0 0

Properties - Space

$$EF(S[0,n)) = n\left\lceil \log\frac{u}{n} \right\rceil$$

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n ones

$$EF(S[0,n)) = n\left\lceil \log \frac{u}{n} \right\rceil$$

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L = 01110011110111011110101 1

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n ones

We store a 0 whenever we change bucket.

Properties - Space

$$EF(S[0,n)) = n\left\lceil \log \frac{u}{n} \right\rceil$$

$\lceil \log(u/n) \rceil$

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We store a 0 whenever we change bucket.

n ones

$2^{\lfloor \log n \rfloor}$ zeros

Properties - Space

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$$EF(S[0,n)) = n\left\lceil \log \frac{u}{n} \right\rceil + 2n \text{ bits}$$

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Information Theoretic Lower Bound

The minimum number of bits needed to describe a set $\mathcal{X}$ is $\lceil \log|\mathcal{X}| \rceil$ bits.

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The minimum number of bits needed to describe a set $\mathcal{X}$ is

$$\left\lceil \log |\mathcal{X}| \right\rceil \text{ bits.}$$

$\mathcal{X}$ is the set of all monotone sequence of length n drawn from a universe u.

$$|\mathcal{X}|\ ?$$

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0000000000000000000

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$|\mathcal{X}|$?

000**1**000000000000000

3

$$EF(S[0,n)) = n\left\lceil \log \frac{u}{n} \right\rceil + 2n \text{ bits}$$

Is it good or not?

Information Theoretic Lower Bound

The minimum number of bits needed to describe a set $\mathcal{X}$ is $\left\lceil \log |\mathcal{X}| \right\rceil$ bits.

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3 6

Properties - Space

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3 6 10

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000**1**00**1**000**11**000000

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000**1**00**1**000**11**00000**1**

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With possible repetitions!
(*weak* monotonicity)

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0001001000110000001

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$$|\mathcal{X}| = \binom{u+n}{n}$$

$$\left\lceil \log \binom{u+n}{n} \right\rceil \approx n \log \frac{u+n}{n}$$

000**1**00**1**000**11**00000**1**

3 6 10 11 17

With possible repetitions!
(*weak* monotonicity)

Properties - Space

$$EF(S[0,n)) = \boxed{n\left\lceil \log\frac{u}{n}\right\rceil + 2n \text{ bits}}$$

Is it good or not?

(less than half a bit away [Elias-1974])

Information Theoretic Lower Bound

The minimum number of bits needed to describe a set $\mathcal{X}$ is $\lceil \log|\mathcal{X}| \rceil$ bits.

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optimal

000**1**00**1**000**11**00000**1**

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With possible repetitions!
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`Access` to each S[i] in O(1) worst-case

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queries in $O\left(\log \frac{u}{n}\right)$ worst-case

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`Rank/Select` on bitmaps

Definition

Given a bitvector B of n bits:

$\mathtt{Rank}_{0/1}(\mathtt{i})$ = # of 0/1 in B[0,$\mathtt{i}$)

$\mathtt{Select}_{0/1}(\mathtt{i})$ = position of $\mathtt{i}$-th 0/1

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Examples

B = 1010110101011110101110101

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Examples

B = 1010110101011110101 10101

$\texttt{Rank}_0(5) = 2$

Rank/Select on bitmaps

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Examples

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$\mathtt{Rank}_{1}(7) = 4$

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Examples

B = 1010110101011110101101 01

$\mathtt{Rank}_0(5) = 2$ $\mathtt{Select}_0(5) = 10$

$\mathtt{Rank}_1(7) = 4$

`Rank/Select` on bitmaps

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Examples

B = 1010110101011110101101

$\texttt{Rank}_0(5) = 2$ $\texttt{Select}_0(5) = 10$

$\texttt{Rank}_1(7) = 4$ $\texttt{Select}_1(7) = 11$

Random Access

$S = [3, 4, 7, 13, 14, 15, 21, 43]$

1 2 3 4 5 6 7 8

Random `Access`

S = [3, 4, 7, 13, 14, 15, 21, 43]

1 2 3 4 5 6 7 8

`Access(4)` = S[4] = ?

Random Access

$S = [3, 4, 7, 13, 14, 15, 21, 43]$

1 2 3 4 5 6 7 8

Access(4) $= S[4] = ?$

$H = 1110111010001000$

$L = \underbrace{011}_{r = \lceil \log(u/n) \rceil}100111011101111101011$

Random Access

$S = [3, 4, 7, 13, 14, 15, 21, 43]$

1 2 3 4 5 6 7 8

Access(4) = S[4] = ?

Recall: we store a 0 whenever we change bucket.

H = 1110111010001000

L = 011100111101110111101011

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$r = \lceil \log(u/n) \rceil$

Access(i) = Select₁(i)

Random Access

S = [3, 4, 7, 13, 14, 15, 21, 43]

1 2 3 4 5 6 7 8

Access(4) = S[4] = ?

Recall: we store a 0 whenever we change bucket.

H = 1110111010001000

L = 011100111101110111101011

$r = \lceil \log(u/n) \rceil$

$\text{Access}(i) = \text{Rank}_0(\text{Select}_1(i))$

Random Access

S = [3, 4, 7, 13, 14, 15, 21, 43]

1 2 3 4 5 6 7 8

Access(4) = S[4] = 001000

Recall: we store a 0 whenever we change bucket.

H = 11101110100010000

L = 011100111011101111010111

$r = \lceil \log(u/n) \rceil$

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S = [3, 4, 7, 13, 14, 15, 21, 43]

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Access(4) = S[4] = 001000

Recall: we store a 0 whenever we change bucket.

H = 11101 11010001000

L = 011100111101110111101011

$r = \lceil \log(u/n) \rceil$

$$\text{Access}(i) = \text{Rank}_0(\text{Select}_1(i)) = \text{Select}_1(i) - i$$

Random Access

$S = [3, 4, 7, 13, 14, 15, 21, 43]$

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L = 011100111101110111101011

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```
Access(i) = Select₁(i) - i << r | L[(i-1)r,ir)
```

Random Access

S = [3, 4, 7, 13, 14, 15, 21, 43]

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Access(4) = S[4] = 001 101

Recall: we store a 0 whenever we change bucket.

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Access(4) = S[4] = 001101

Access(7) = S[7] = ?

Recall: we store a 0 whenever we change bucket.

H = 1110111010001000

L = 011100111101110111101011

$r = \lceil \log(u/n) \rceil$

`Access(i) = Select`$_1$`(i) - i << r | L[(i-1)r,ir)`

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Complexity: O(1)

Access(i) = Select$_1$(i) - i << r | L[(i-1)r,ir)

Available Implementations

Library	Author(s)	Link	Language
folly	Facebook, Inc.	https://github.com/facebook/folly	C++
sdsl	Simon Gog	https://github.com/simongog/sdsl-lite	C++
ds2i	Giuseppe Ottaviano Rossano Venturini Nicola Tonellotto	https://github.com/ot/ds2i	C++
Sux	Sebastiano Vigna	http://sux.di.unimi.it	Java/C++

Killer applications

1. Inverted Indexes

Killer applications

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2. Social Networks

Killer applications

1. Inverted Indexes

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Unicorn: A System for Searching the Social Graph

Michael Curtiss, Iain Becker, Tudor Bosman, Sergey Doroshenko,
Lucian Grijincu, Tom Jackson, Sandhya Kunnatur, Soren Lassen, Philip Pronin,
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Facebook, Inc.

ABSTRACT

Unicorn is an online, in-memory social graph-aware indexing system designed to search trillions of edges between tens of billions of users and entities on thousands of commodity servers. Unicorn is based on standard concepts in informa-

rative of the evolution of Unicorn's architecture, as well as documentation for the major features and components of the system.

To the best of our knowledge, no other online graph retrieval system has ever been built with the scale of Unicorn

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Open Source

All Unicorn index server and aggregator code is written in C++. Unicorn relies extensively on modules in Facebook's "Folly" Open Source Library [5]. As part of the effort of releasing Graph Search, we have open-sourced a C++ implementation of the Elias-Fano index representation [31] as part of Folly.

Killer applications

1. Inverted Indexes

2. Social Networks

3. Compressed Tries for N-Grams

N-grams - Introduction

Strings of *N* words.

N typically ranges from 1 to 5.

Extracted from text using a *sliding window* approach.

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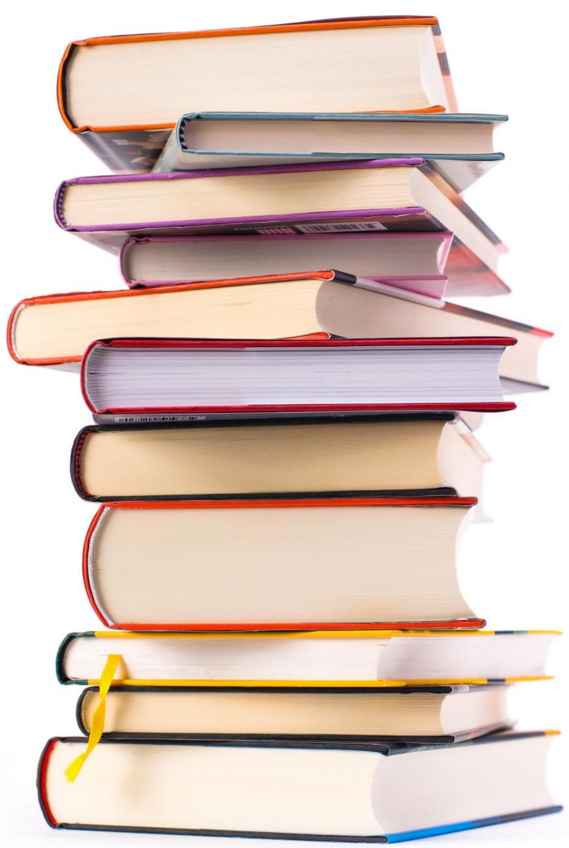

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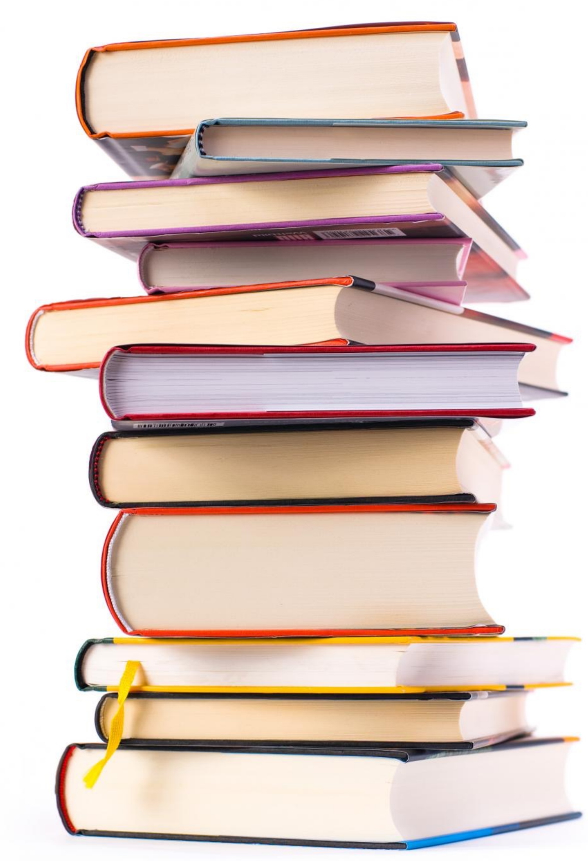

Google Books

≈ 6% of the books ever published

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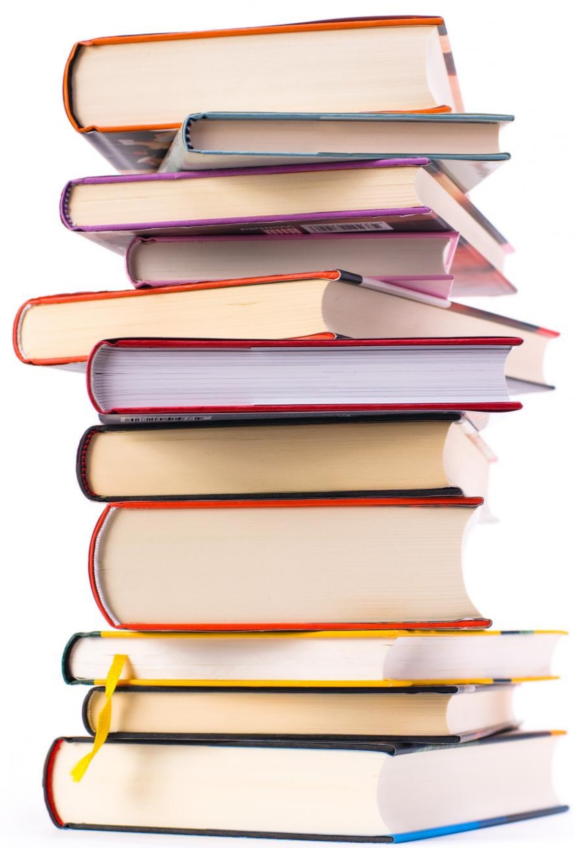

Google Books

≈ 6% of the books ever published

N	number of grams
1	24,359,473
2	667,284,771
3	7,397,041,901
4	1,644,807,896
5	1,415,355,596

More than 11 billion grams.

N-grams - Challenge

Store massive *N*-grams datasets in **compressed space** such that given a pattern, we can **return its value efficiently**.

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N-Gram **values**

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frequency count
(integer)

N-Gram **values**

N-grams - Challenge

Store massive *N*-grams datasets in **compressed space** such that given a pattern, we can **return its value efficiently**.

N-Gram **values** → frequency count (integer)

N-Gram **values** → probability weight (floating point)

N-grams - Challenge

Store massive *N*-grams datasets in **compressed space** such that given a pattern, we can **return its value efficiently**.

N-Gram **values**

→ frequency count (integer)

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Efficient **map**

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Efficient **map** → **hash**
+ time
- space

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Efficient **map**

hash
+ time
- space

trie
+ space
- time

N-grams - Challenge

Store massive *N*-grams datasets in **compressed space** such that given a pattern, we can **return its value efficiently**.

N-Gram **values** → frequency count (integer)

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Efficient **map** → **hash**: + time, - space

Efficient **map** → **trie**: + space, - time

Active field of research
Many software libraries

- **KenLM** [Heafield, WMT 2011]
- **BerkeleyLM** [Pauls and Klein, ACL 2011]
- **ExpGram** [Watanabe at el., IJCNLP 2009]
- **IRSTLM** [Federico et al., ACL 2008]
- **RandLM** [Talbot and Osborne, ACL 2007]
- **SRILM** [Stolcke, INTERSPEECH 2002]

Trie Indexing

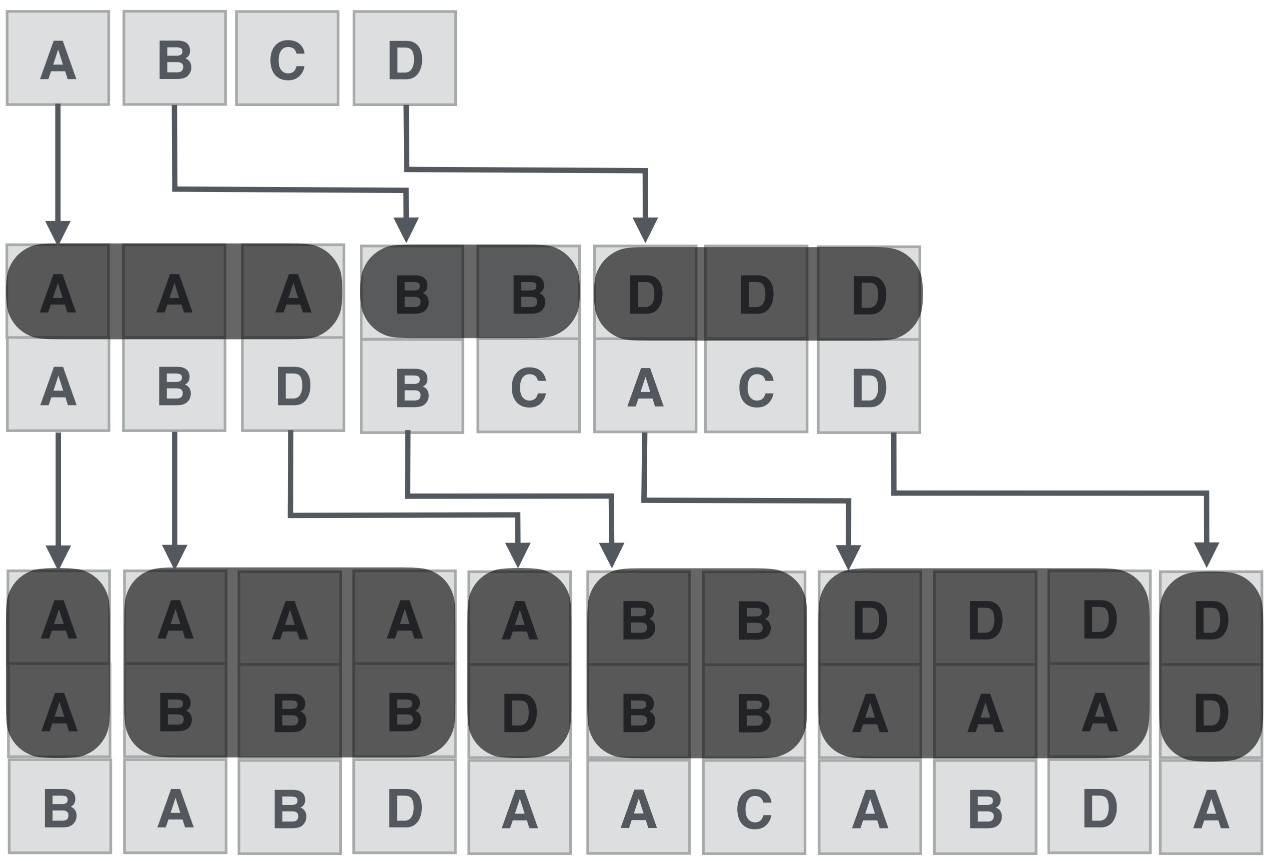
Trie Indexing

A	B	C	D

A	A	A	B	B	D	D	D
A	B	D	B	C	A	C	D

A	A	A	A	A	B	B	D	D	D	D
A	B	B	B	D	B	B	A	A	A	D
B	A	B	D	A	A	C	A	B	D	A

Trie Indexing



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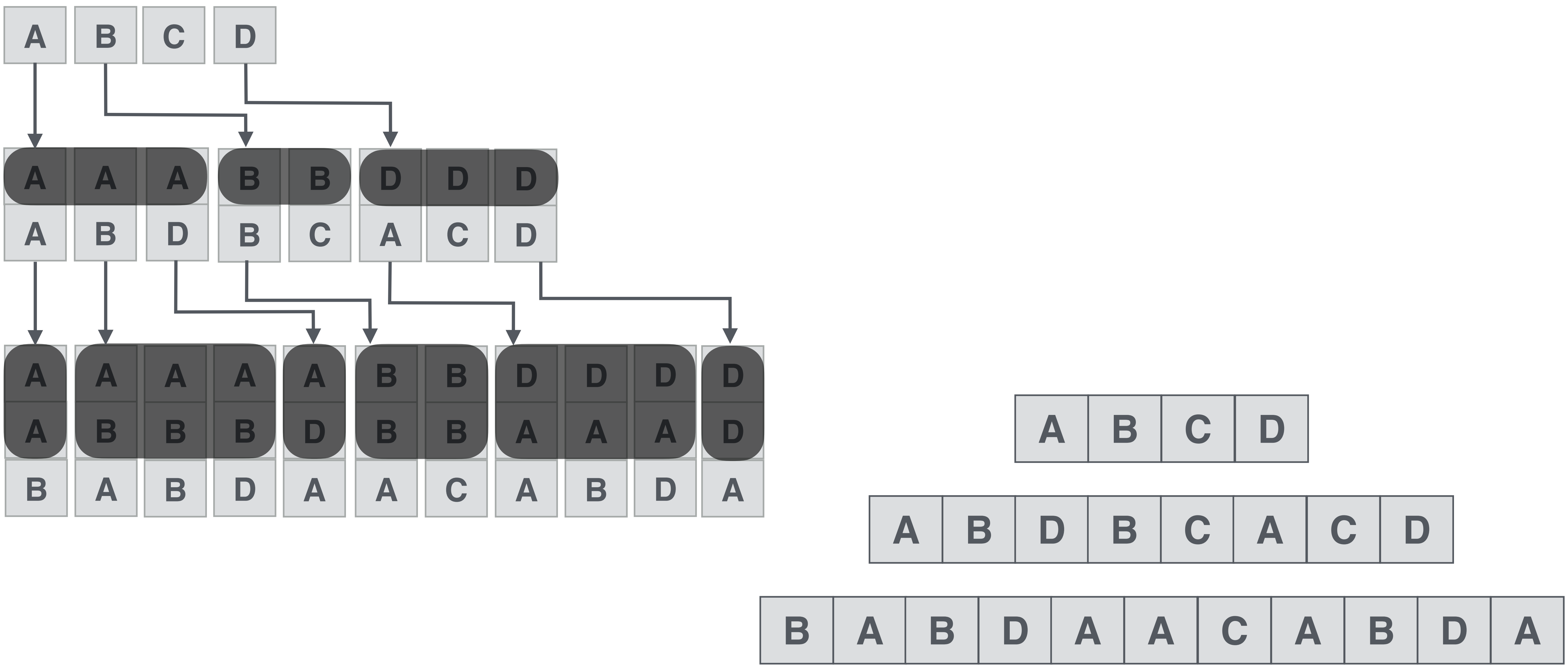

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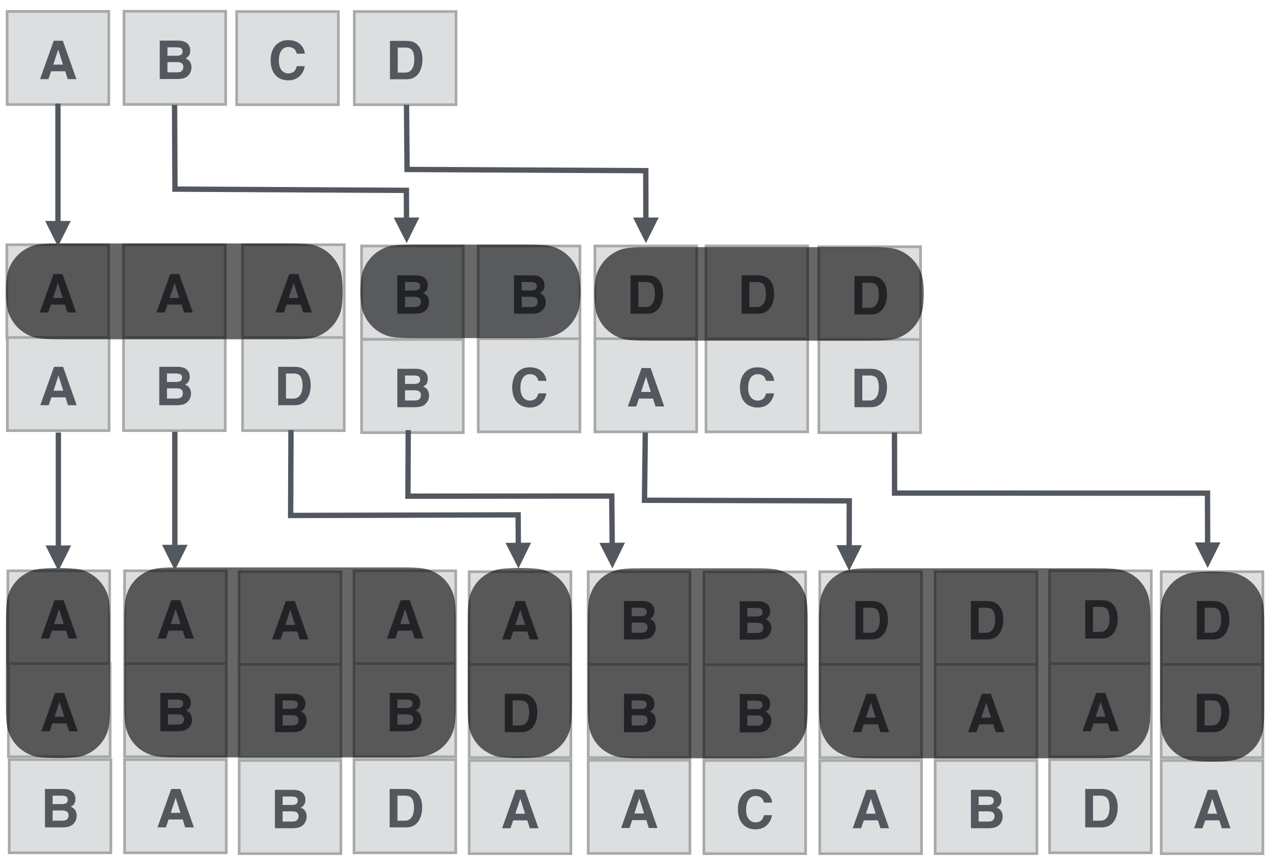

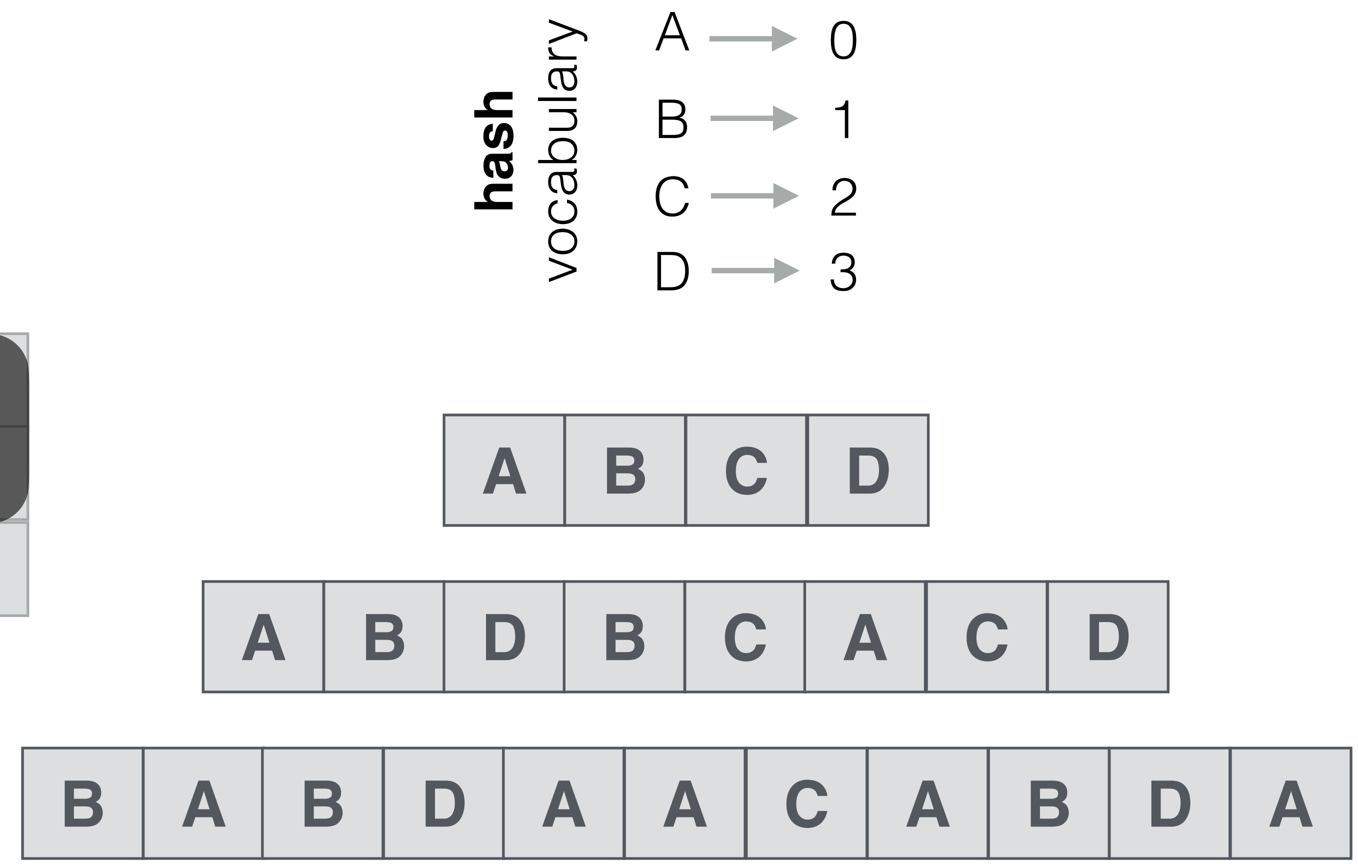

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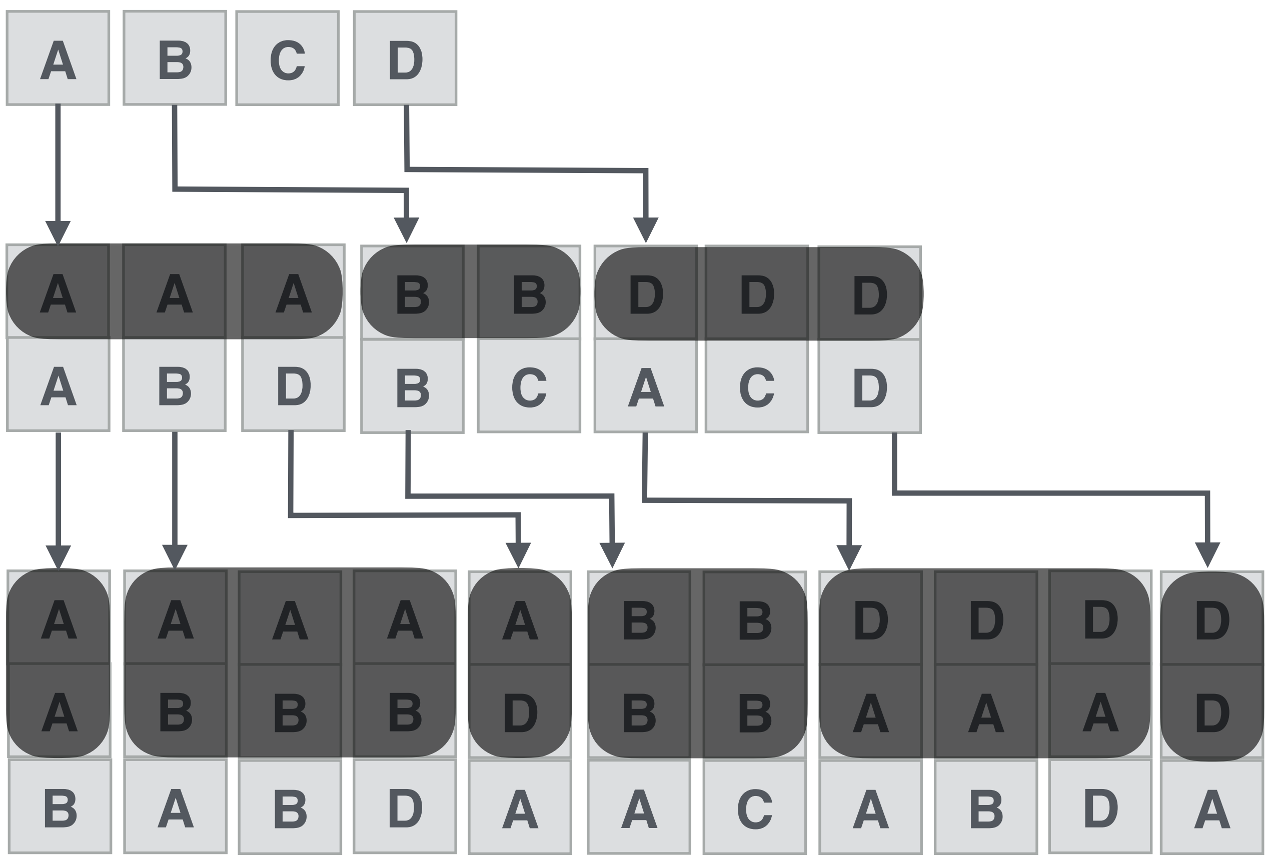

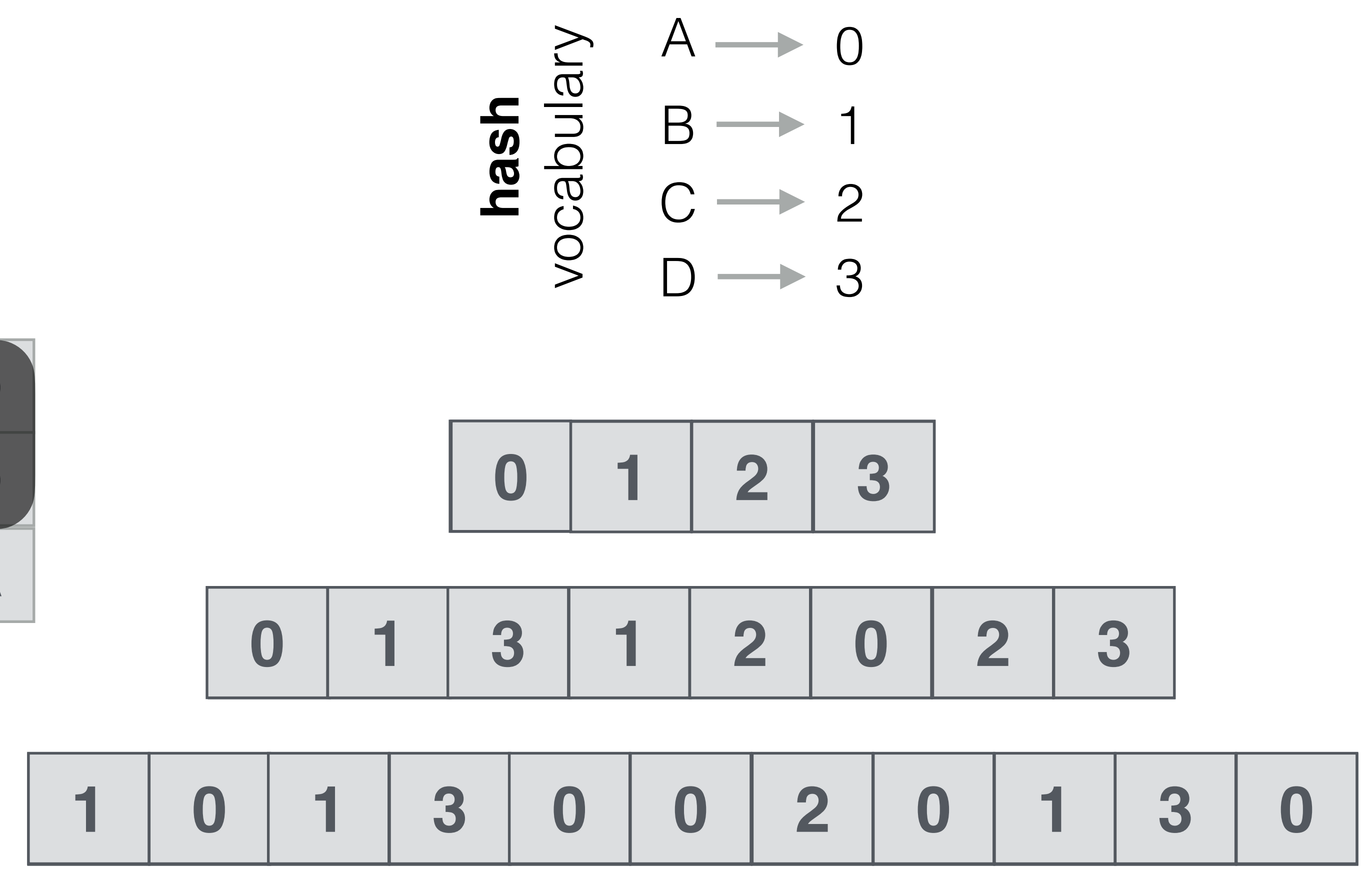

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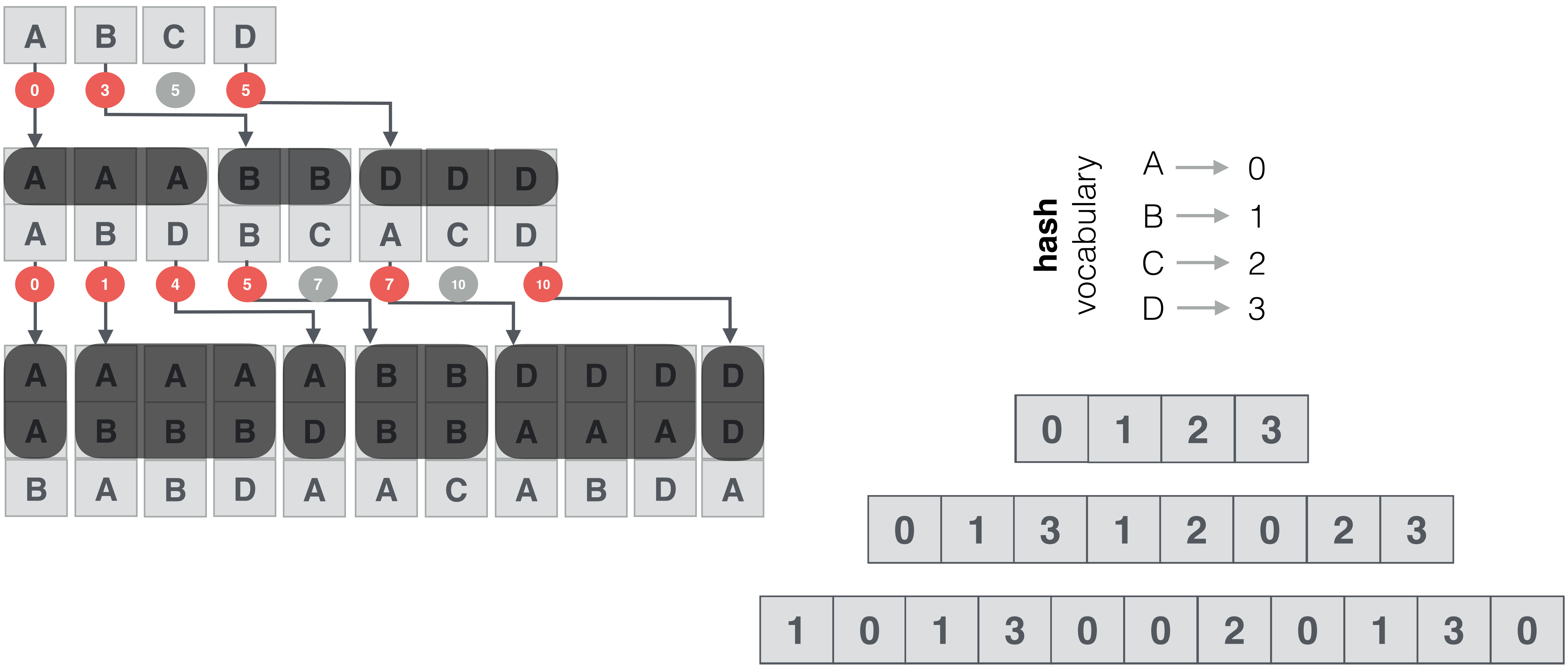

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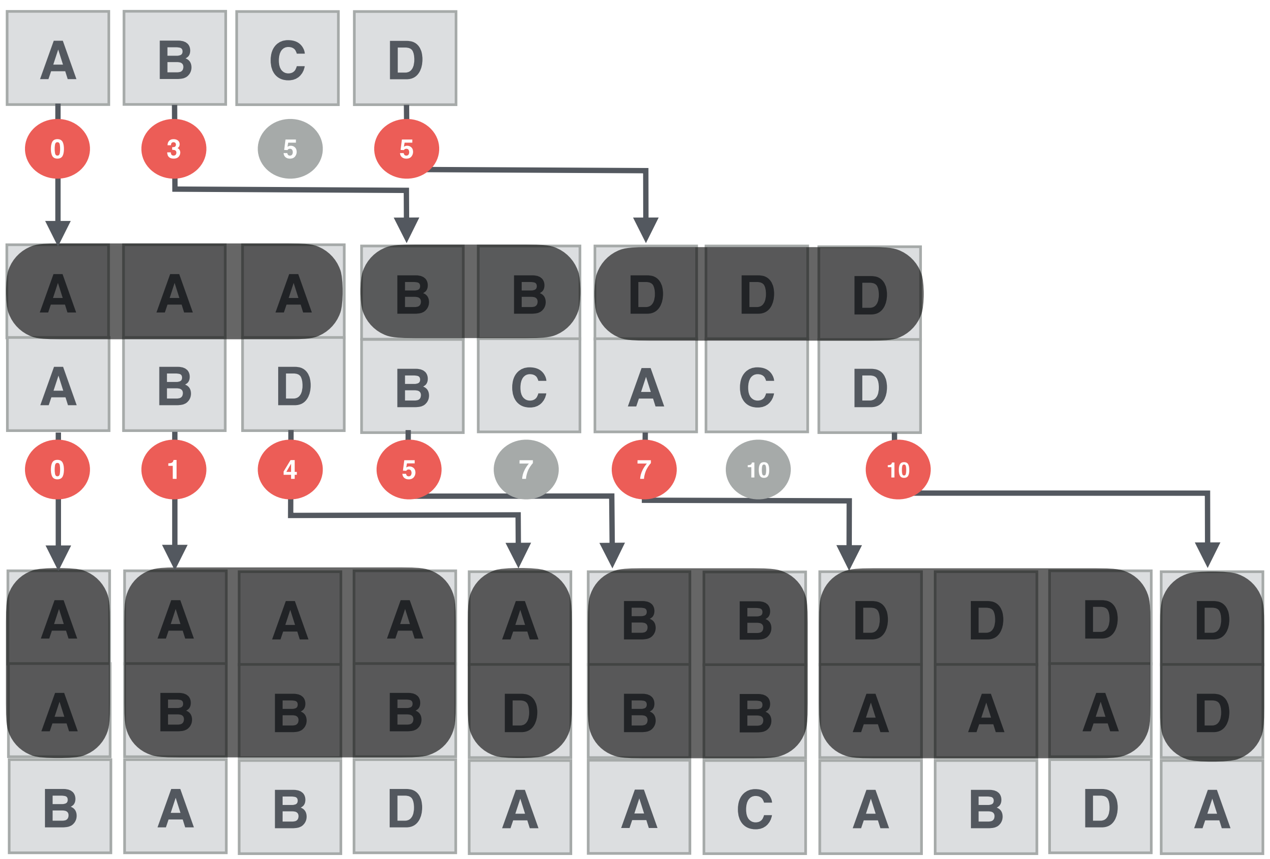

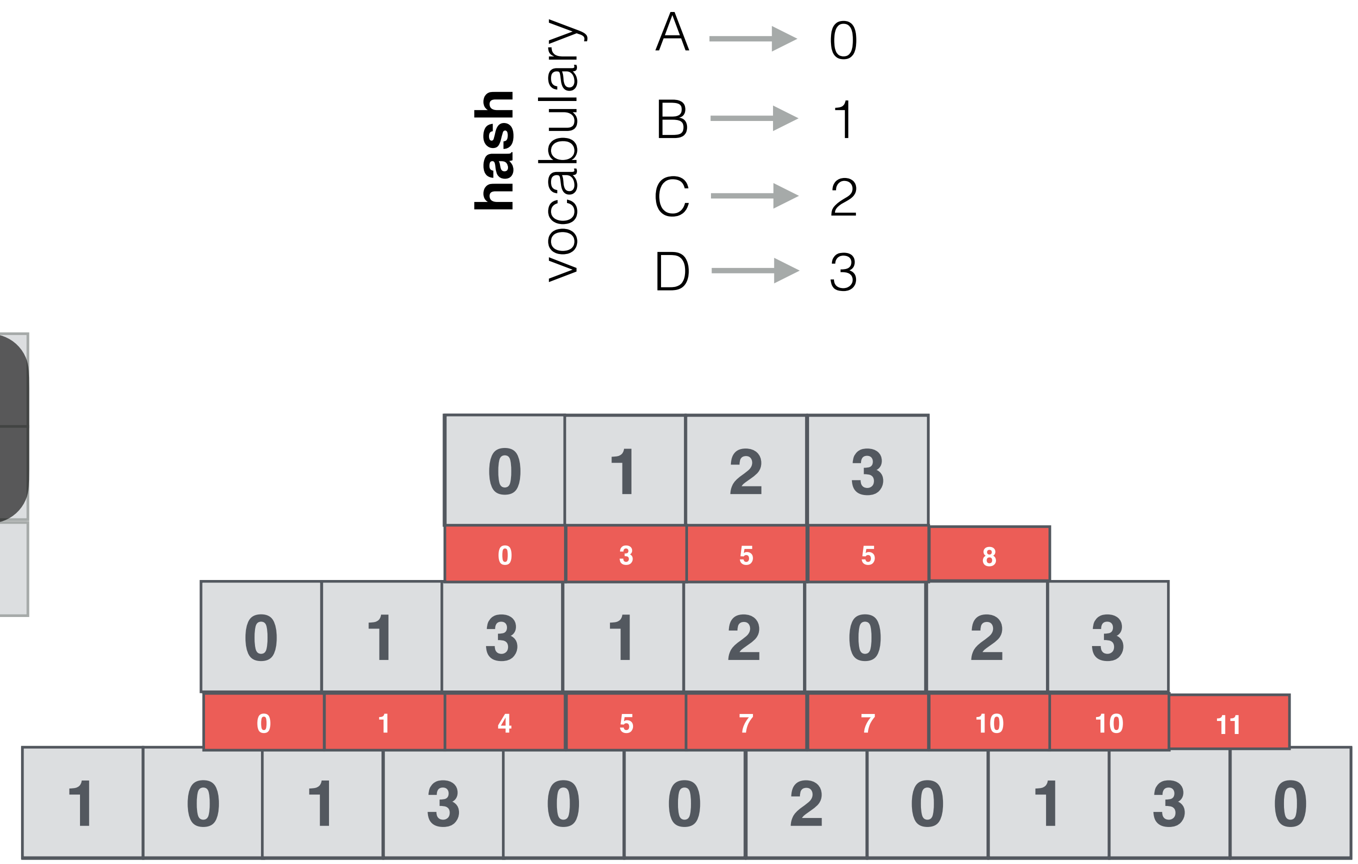

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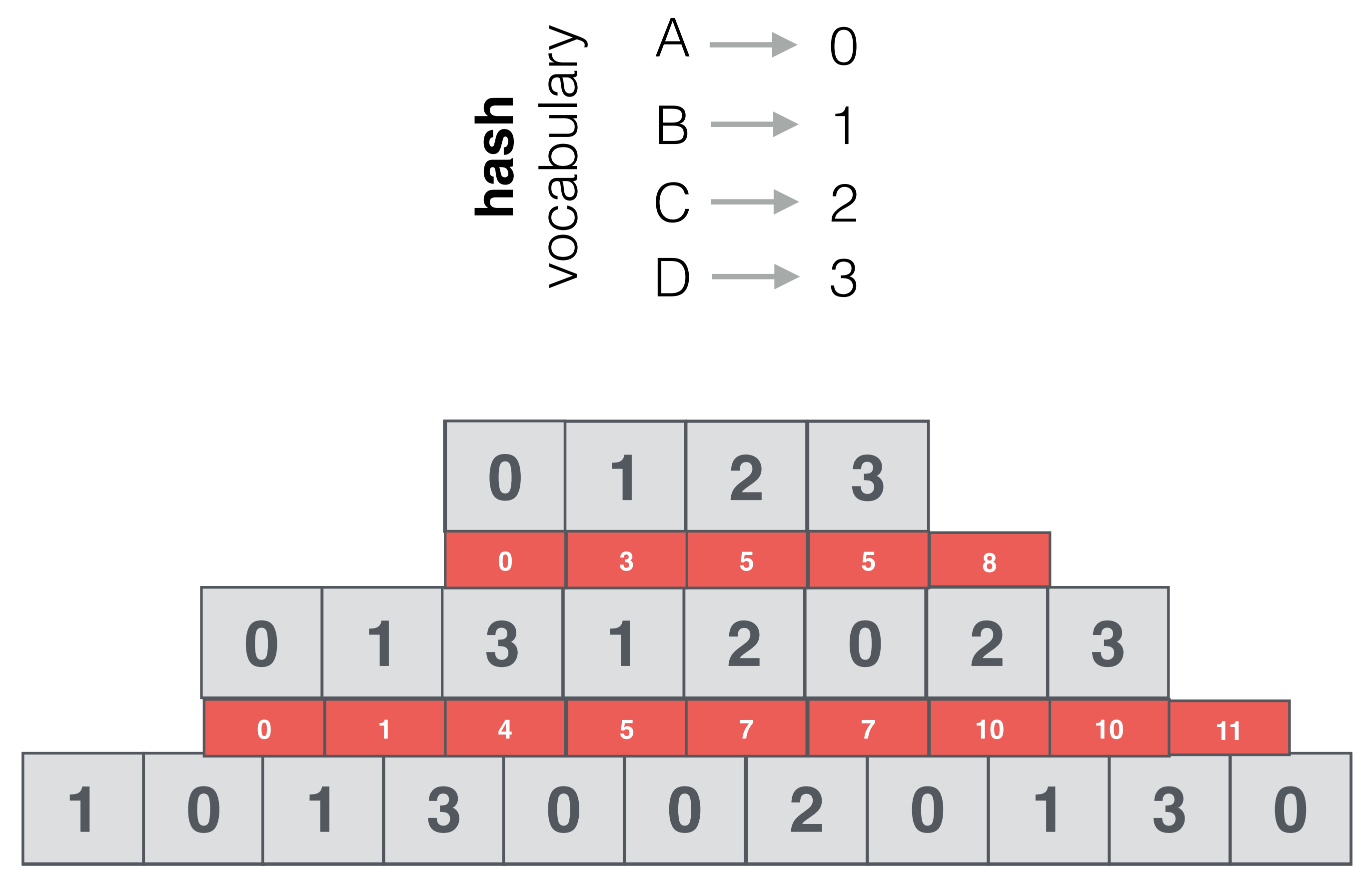
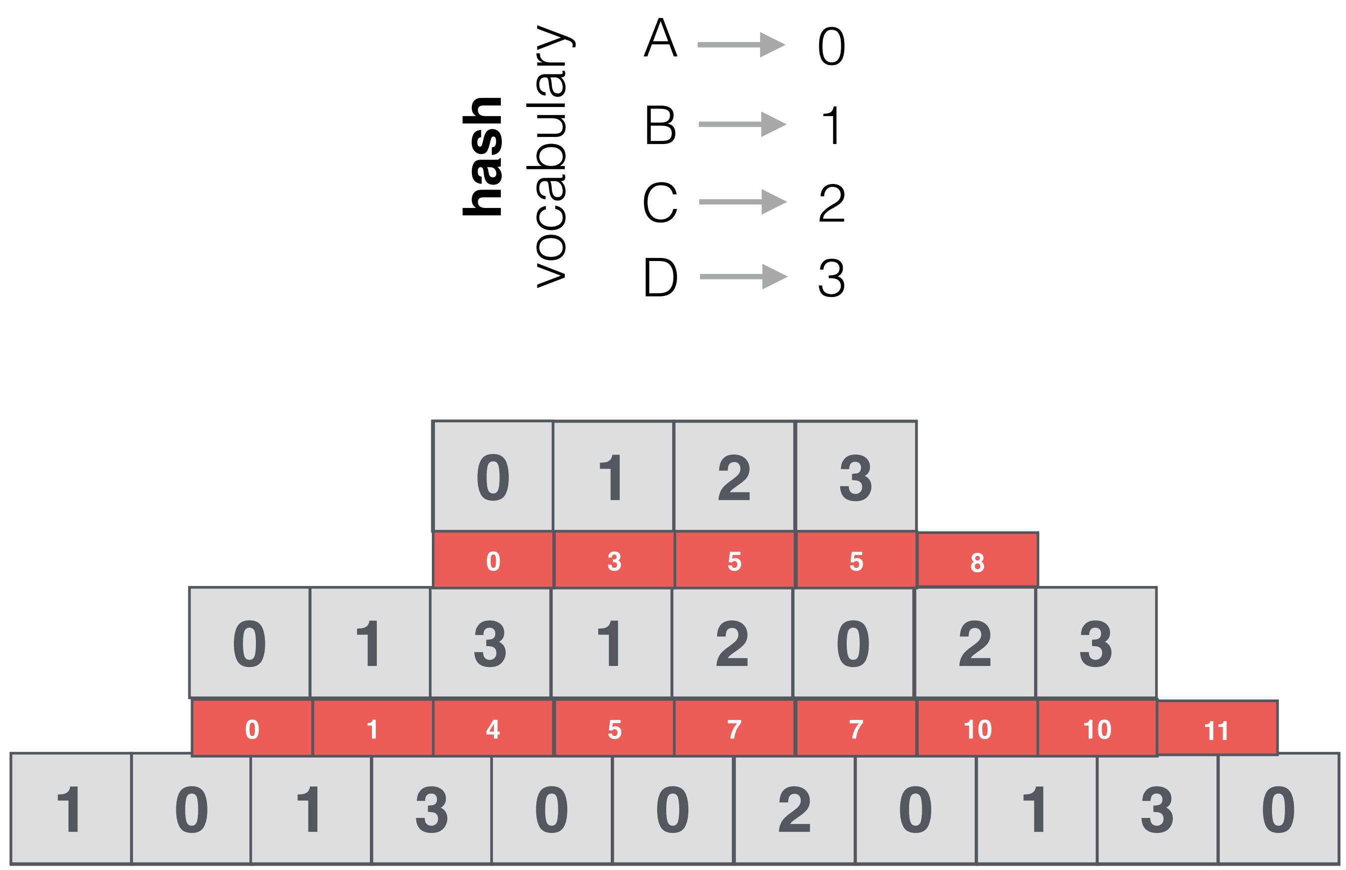


Trie Indexing

We need an encoder for integer sequences, supporting fast **random** Access.



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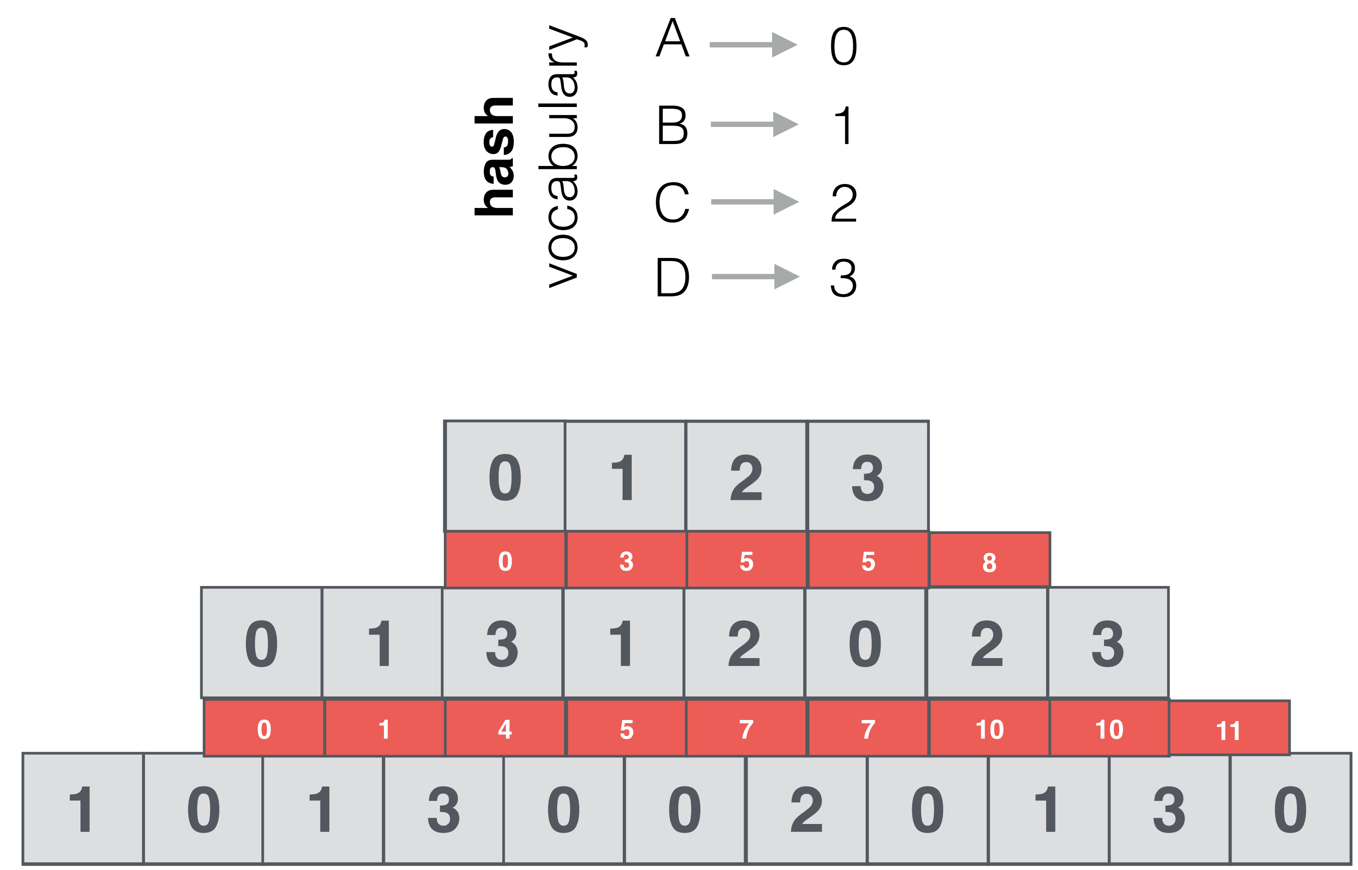
We need an encoder for integer sequences, supporting fast **random** Access.

Take *range-wise* prefix sums on gram-ID sequences.



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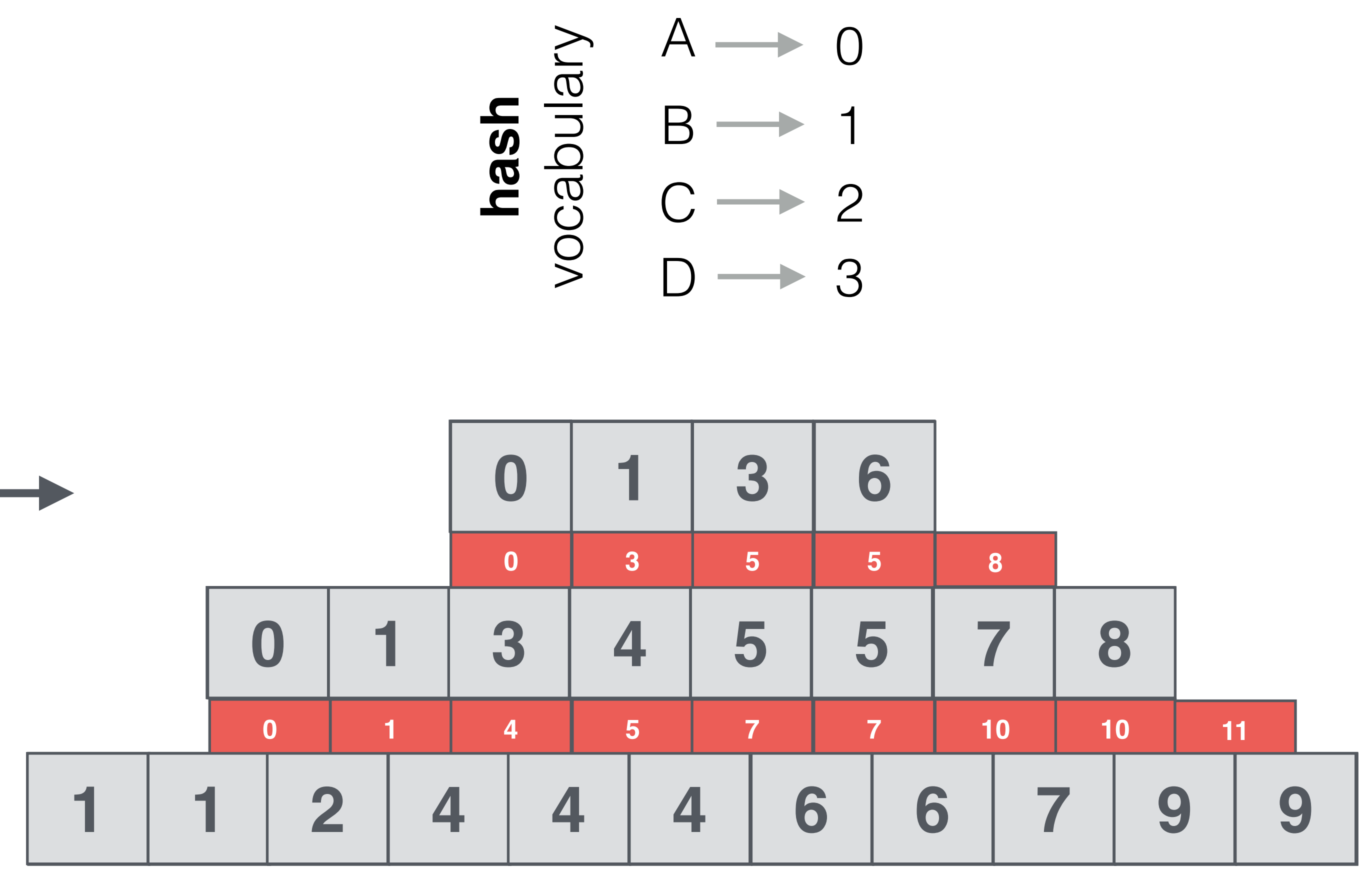

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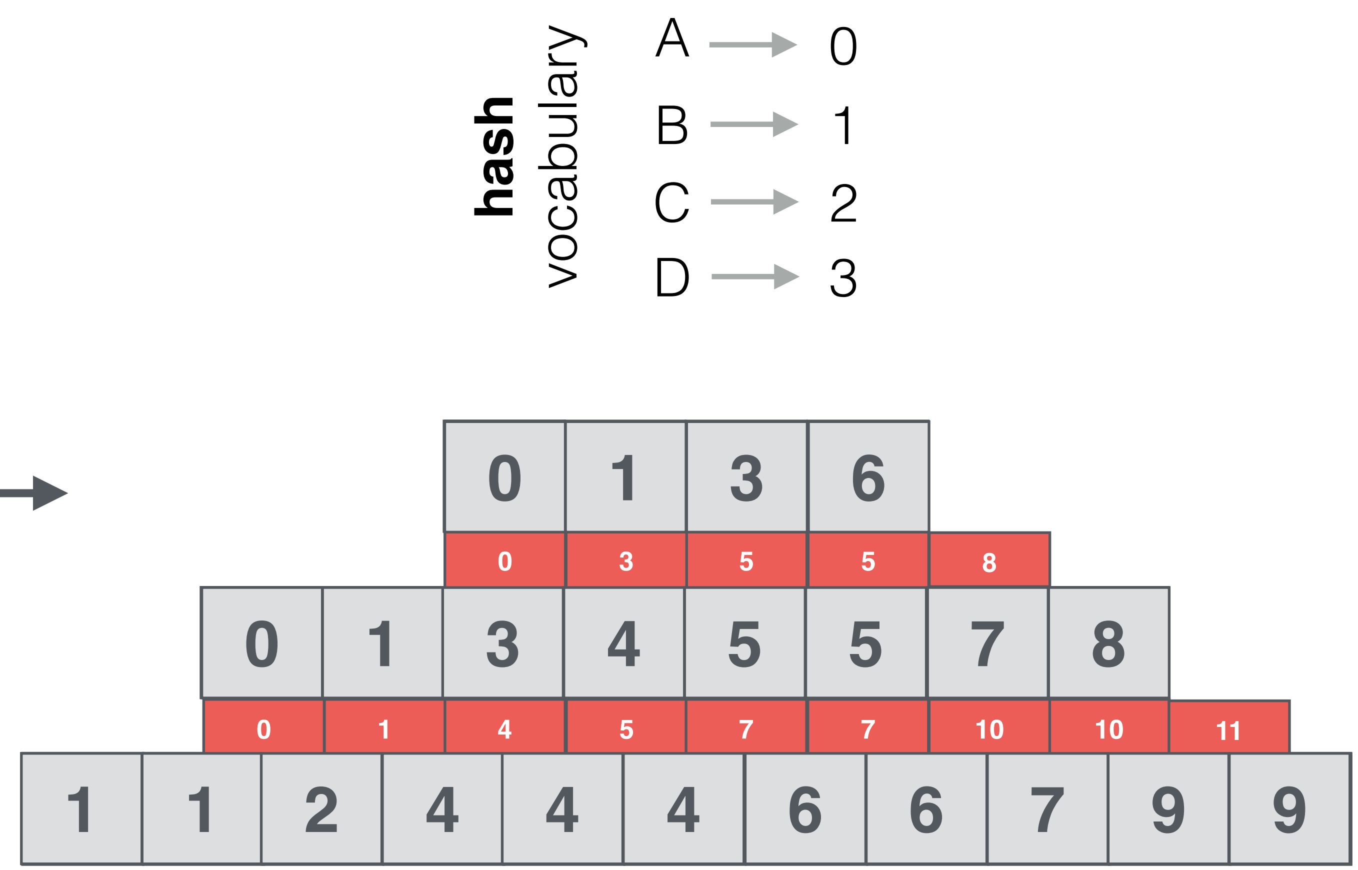

Elias-Fano Tries

One Successor query per level

Constant-time random Access

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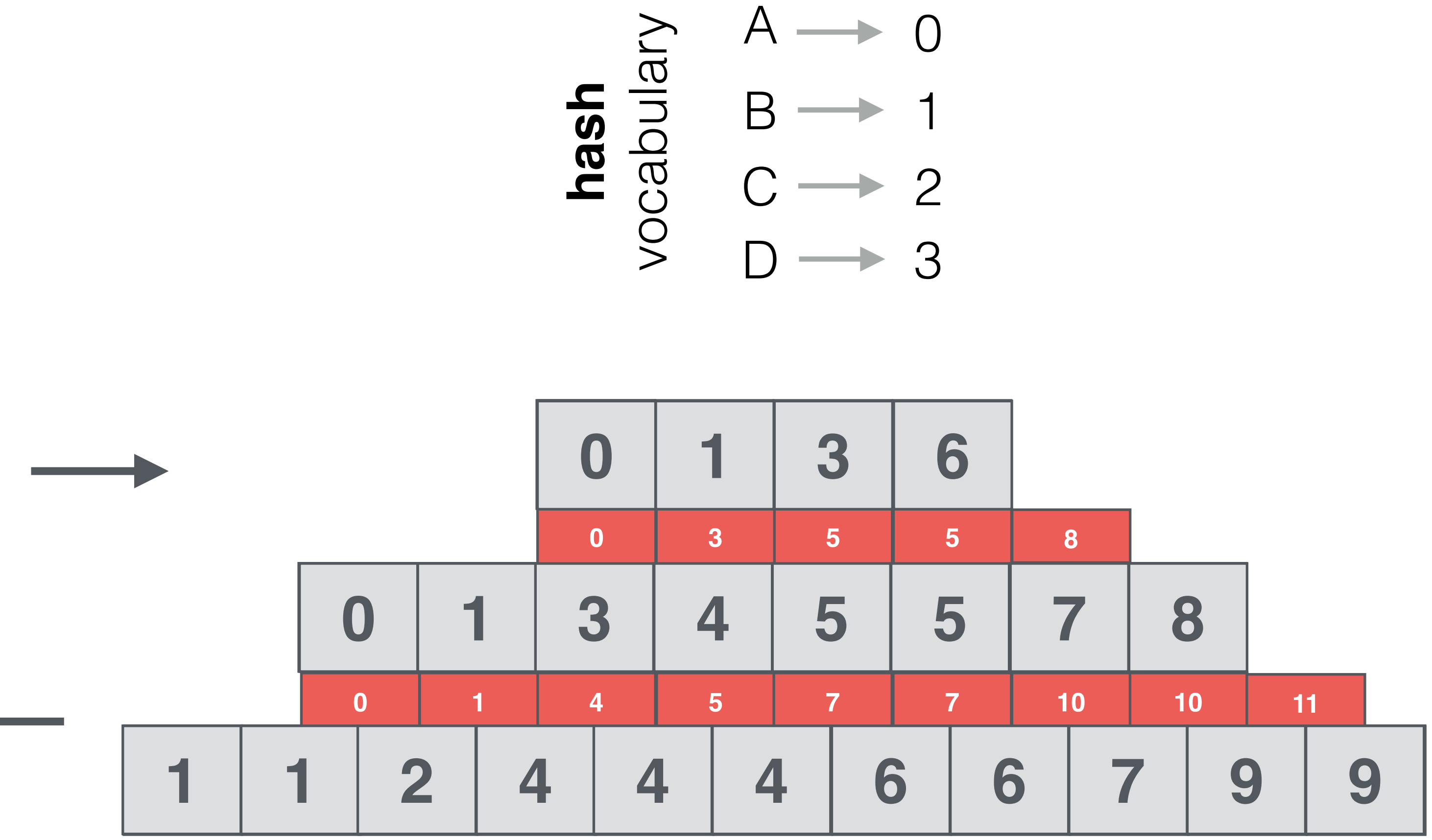

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Remember:
Elias-Fano takes
log(u/n) + 2 bits
per integer

Context-based ID Remapping

Observation: the number of words following a given context is **small**.

High-level idea: map a word ID to the **position** it takes within its *sibling* IDs (the IDs following a context of fixed length k).

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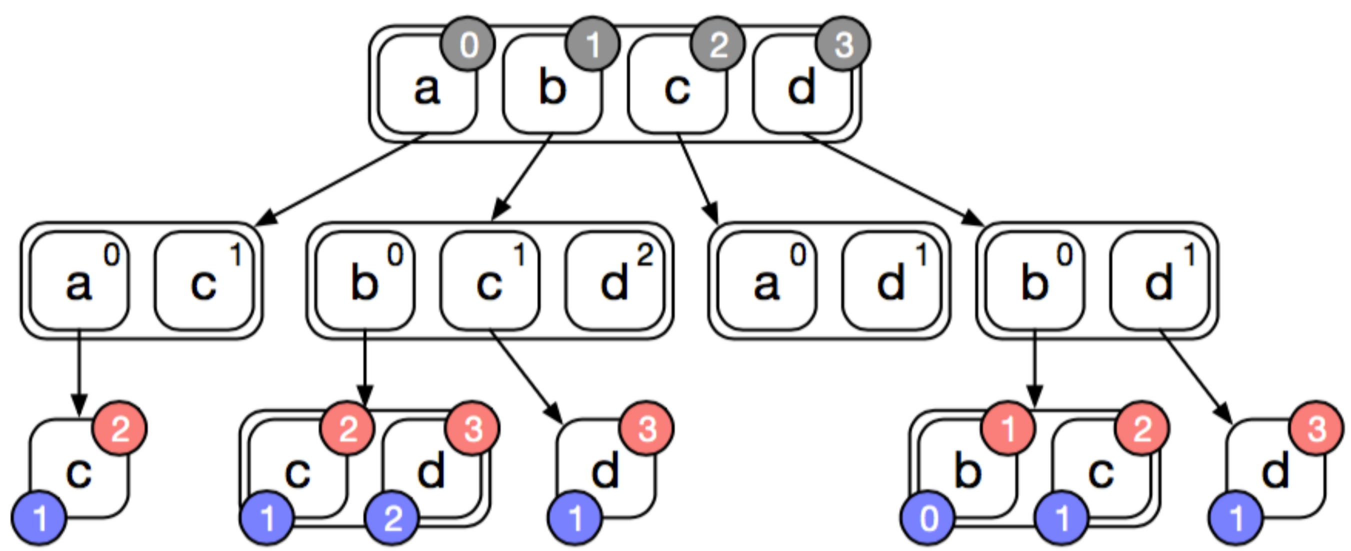

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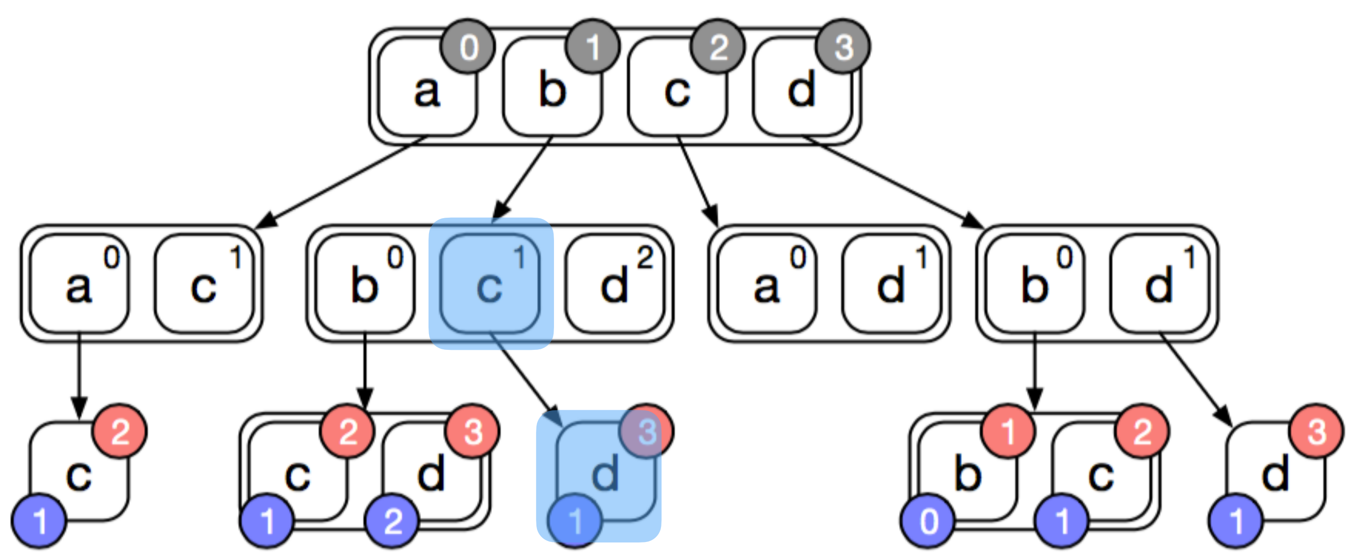

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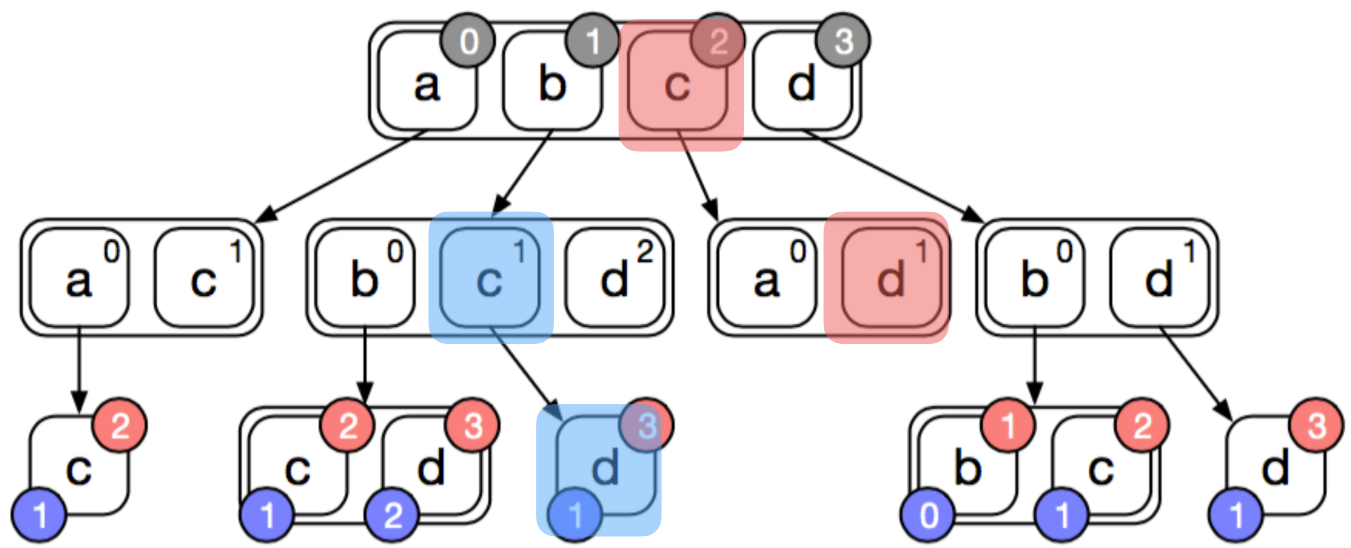

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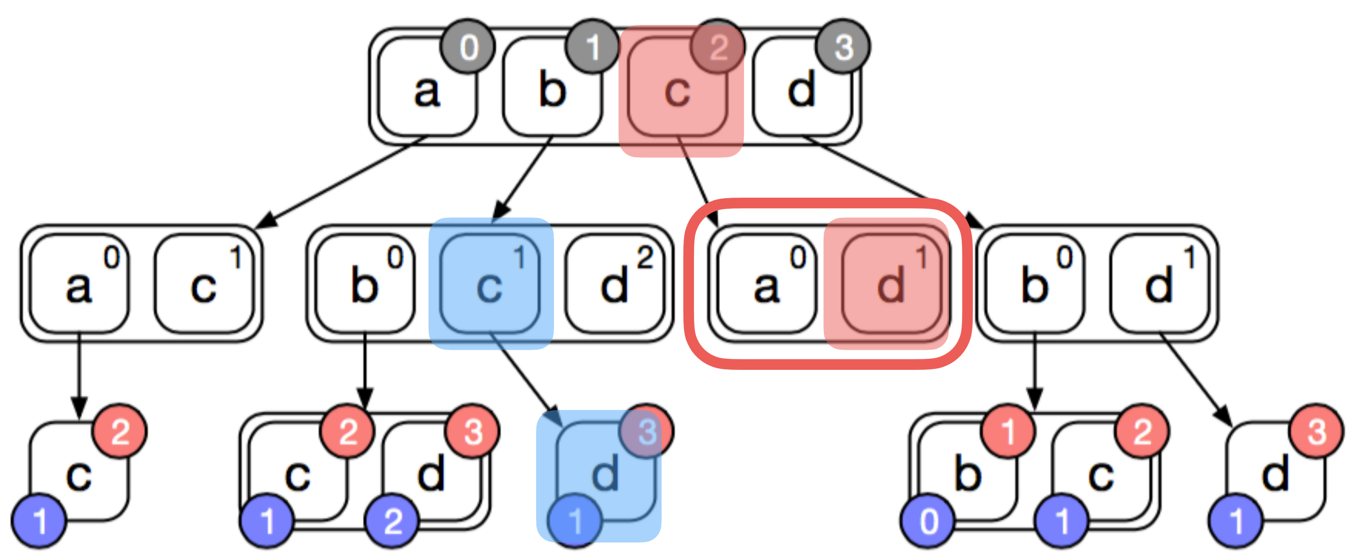

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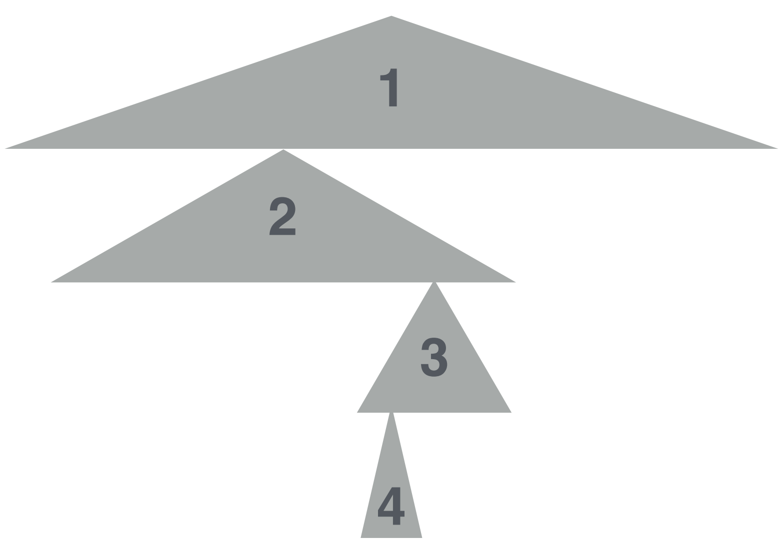

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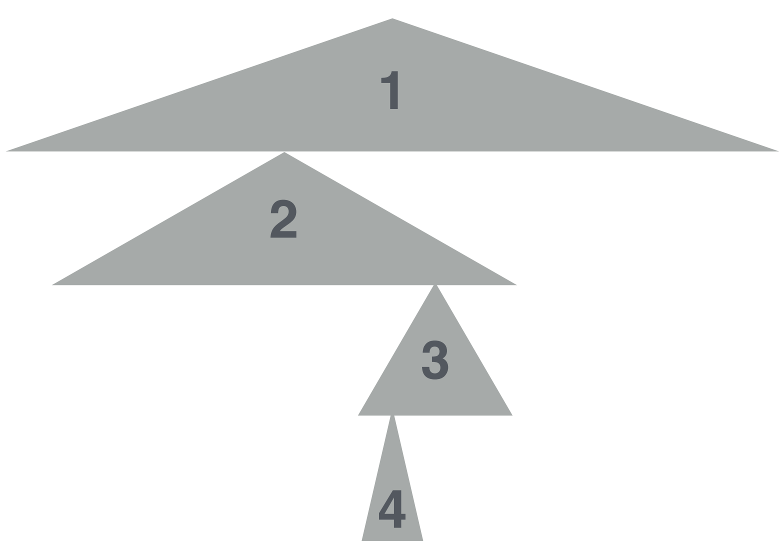

u/n by varying context-length *k*

	k	3-grams	4-grams	5-grams
Europarl	0	2404	2782	2920
	1	213 (×11.28)	480 (×5.79)	646 (×4.52)
	2	2404	48 (×57.95)	101 (×28.91)
YahooV2	0	7350	7197	7417
	1	753 (×9.76)	1461 (×4.93)	1963 (×3.78)
	2	7350	104 (×69.20)	249 (×29.79)
GoogleV2	0	4050	6631	6793
	1	1025 (×3.95)	2192 (×3.03)	2772 (×2.45)
	2	4050	221 (×30.00)	503 (×13.50)

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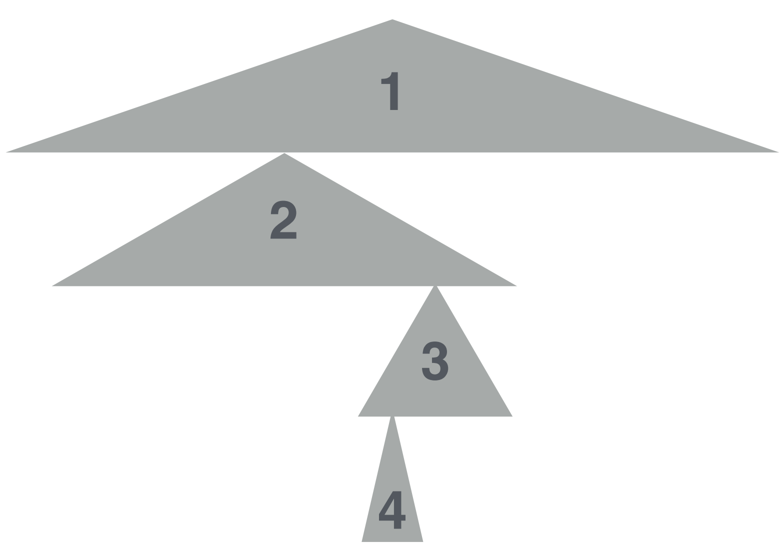

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Experimental Analysis - EF/PEF (R)Trie

N	Europarl	YahooV2	GoogleV2
	n	n	n
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Total	101 428 257	828 223 677	11 131 242 087
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Test machine
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C++ implementation
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	PEF	1.87 (−4.99%)	1.35 (+5.93%)	1.91 (−12.03%)	1.73 (+8.00%)	1.52 (−28.60%)	1.91 (−8.79%)
CONTEXT-BASED ID REMAPPING $k = 1$	EF	1.67 (−15.30%)	1.58 (+23.86%)	1.89 (−12.92%)	2.05 (+28.07%)	1.91 (−10.24%)	3.03 (+44.61%)
	PEF	1.53 (−22.36%)	1.61 (+25.89%)	1.63 (−24.91%)	2.16 (+35.22%)	1.31 (−38.71%)	2.30 (+9.88%)
CONTEXT-BASED ID REMAPPING $k = 2$	EF	1.46 (−25.62%)	1.60 (+25.17%)	1.68 (−22.32%)	2.08 (+30.23%)	—	—
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- reduces space by more than **36%** on average → **you will** notice this**!**
- brings approximately **30%** more time → **will you** notice this**?**

Experimental Analysis - Overall comparison

	Europarl		YahooV2		GoogleV2	
	bpg	μs × query	bpg	μs × query	bpg	μs × query
PEF-Trie	1.87	1.35	1.91	1.73	1.52	1.91
PEF-RTrie	1.28	1.64	1.38	2.15	1.31	2.30
BerkeleyLM C.	1.70 (−8.89%) (+32.90%)	2.83 (+108.88%) (+72.70%)	1.69 (−11.41%) (+22.04%)	3.48 (+101.84%) (+61.70%)	1.45 (−4.87%) (+10.83%)	4.13 (+116.57%) (+79.76%)
BerkeleyLM H.3	6.70 (+258.81%) (+423.40%)	0.97 (−28.46%) (−40.85%)	7.82 (+310.38%) (+465.36%)	1.13 (−34.35%) (−47.41%)	9.24 (+507.79%) (+608.07%)	2.18 (+13.95%) (−5.42%)
BerkeleyLM H.50	7.96 (+326.03%) (+521.45%)	0.97 (−28.49%) (−40.88%)	9.37 (+391.32%) (+576.87%)	0.96 (−44.27%) (−55.35%)	—	—
Expgram	2.06 (+10.18%) (+60.73%)	2.80 (+106.61%) (+70.82%)	2.24 (+17.36%) (+61.68%)	9.23 (+435.33%) (+328.87%)	—	—
KenLM T.	2.99 (+60.11%) (+133.56%)	1.28 (−5.47%) (−21.84%)	3.44 (+80.39%) (+148.52%)	1.94 (+12.32%) (−10.01%)	—	—
Marisa	3.61 (+93.09%) (+181.66%)	2.06 (+52.00%) (+25.67%)	3.81 (+99.60%) (+174.98%)	3.24 (+87.96%) (+50.58%)	—	—
RandLM	1.81 (−3.06%) (+41.41%)	4.39 (+224.20%) (+168.04%)	2.02 (+6.18%) (+46.29%)	5.08 (+194.35%) (+135.82%)	2.60 (+70.73%) (+98.90%)	9.25 (+384.54%) (+302.19%)

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Marisa	3.61 (+93.09%) (+181.66%)	2.06 (+52.00%) (+25.67%)	3.81 (+99.60%) (+174.98%)	3.24 (+87.96%) (+50.58%)	—	—
RandLM	1.81 (−3.06%) (+41.41%)	4.39 (+224.20%) (+168.04%)	2.02 (+6.18%) (+46.29%)	5.08 (+194.35%) (+135.82%)	2.60 (+70.73%) (+98.90%)	9.25 (+384.54%) (+302.19%)

Experimental Analysis - Overall comparison

	Europarl		YahooV2		GoogleV2	
	bpg	μs × query	bpg	μs × query	bpg	μs × query
PEF-Trie	1.87	1.35	1.91	1.73	1.52	1.91
PEF-RTrie	1.28	1.64	1.38	2.15	1.31	2.30
BerkeleyLM C.	1.70 (−8.89%) (+32.90%)	2.83 ([illegible]) (+72.70%) 2X	1.69 (−11.41%) (+22.04%)	3.48 ([illegible]) (+61.70%) 2X	1.45 (−4.87%) (+10.83%)	4.13 ([illegible]) (+79.76%) 2X
BerkeleyLM H.3	6.70 (+258.81%) ([illegible]) 2.5÷	0.97 (−28.46%) (−40.85%)	7.82 (+310.38%) ([illegible]) 3.1÷	1.13 (−34.35%) (−47.41%)	9.24 ([illegible]) ([illegible]) 5.5X	2.18 (+13.95%) (−5.42%)
BerkeleyLM H.50	7.96 ([illegible]) (+521.45%) 5.2X	0.97 (−28.49%) (−40.88%)	9.37 ([illegible]) (+576.87%) 5.8X	0.96 (−44.27%) (−55.35%)	—	—
Expgram	2.06 (+10.18%) (+60.73%)	2.80 ([illegible]) (+70.82%) 2X	2.24 (+17.36%) (+61.68%)	9.23 ([illegible]) (+328.87%) 3.5X	—	—
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- Elias-Fano Tries substantially **outperform ALL** previous solutions in **both space and time**.
- As fast as the state-of-the-art (KenLM) but more than twice smaller.

Summary

Elias-Fano encodes *monotone integer sequences* in *space close to the information theoretic minimum*, while allowing *powerful search operations*, namely `Predecessor/Successor` queries and random `Access`.

Successfully applied to crucial problems, such as *inverted indexes, social graphs* and *tries* representation.

Several *optimized* software implementations are available.

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Thanks for your attention, time, patience!

Any questions?

successor **example**

$S = [3, 4, 7, 13, 14, 15, 21, 43]$

1 2 3 4 5 6 7 8

H = 1110111010001000

L = 011100111101110111101011

`successor` example

S = [3, 4, 7, 13, 14, 15, 21, 43]

1 2 3 4 5 6 7 8

`successor(12)` = ?

H = 1110111010001000

L = 011100111101110111101011

`successor` example

S = [3, 4, 7, 13, 14, 15, 21, 43]

1 2 3 4 5 6 7 8

`successor(12)` = ?

001100

H = 1110111010001000

L = 011100111101110111101011

successor **example**

S = [3, 4, 7, 13, 14, 15, 21, 43]

1 2 3 4 5 6 7 8

successor(12) = ?

h_{12} = 001100

H = 1110111010001000

L = 01110011110111011111101011

successor example

$S = [3, 4, 7, 13, 14, 15, 21, 43]$

1 2 3 4 5 6 7 8

successor(12) = ?

$h_{12} = 001100$

$p_1 = select_0(h_x) - h_x$

$p_2 = select_0(h_x + 1) - h_x - 1$

H = 1110111010001000

L = 011100111101110111101011

successor example

$S = [3, 4, 7, 13, 14, 15, 21, 43]$

1 2 3 4 5 6 7 8

$\texttt{successor(12)} = ?$

$h_{12} = 001100$

$p_1 = \texttt{select}_0(h_x) - h_x$

$p_2 = \texttt{select}_0(h_x+1) - h_x - 1$

$H = \cancel{1110}0111010001000$

$L = 011100111011101111101011$

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h_{12} = 001100

```
p1 = select0(hx)-hx
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```

H = ~~1110~~1110~~10001000~~

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$L = \cancel{011100111}101110111\cancel{101011}$

p_1 p_2

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p_1 p_2

binary search
in $[p_1, p_2)$

successor **example**

$S = [3, 4, 7, 13, 14, 15, 21, 43]$

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p_1 p_2

→ *binary search* in $[p_1,p_2)$

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successor(12) = 13

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p_1 p_2

→ *binary search* in $[p_1, p_2)$

Complexity: $O\left(\log \frac{u}{n}\right)$